WEEK 3 LAB

MATH 131: Calculus II
February 6, 2020
Covering Sections 5.2-5.3

Your Name (Print): ANSWER KEY

1. Suppose $\int_1^7 f(x)dx = 28$ and $\int_4^1 f(x)dx = 16$. Evaluate the following integrals being careful to **show each step**. Each step should be the result of **ONE** of the properties of definite integrals as stated in Table 5.4 in Section 5.2.

(a) $\int_4^7 f(x)dx$

$$\int_1^7 f(x)dx = \int_1^4 f(x)dx + \int_4^7 f(x)dx, \text{ so } \int_4^7 f(x)dx = \int_1^7 f(x)dx - \int_1^4 f(x)dx$$

$$\text{and thus } \int_4^7 f(x)dx = \int_1^7 f(x)dx + \int_4^1 f(x)dx = 28 + 16 = 44$$

(b) $\int_7^4 f(x)dx = -\int_4^7 f(x)dx = -44$ (given part (a))

(c) $\int_4^7 [5 - 3f(x)]dx = \int_4^7 5\,dx - \int_4^7 3f(x)dx$

$$= \int_4^7 5\,dx - 3\int_4^7 f(x)dx$$

$= 5(7-4) - 3(44)$ (given part (a))

$= 15 - 132$

$= -117$

2. Evaluate the following integrals. Be sure to show each step or explain your process. BE CAREFUL!

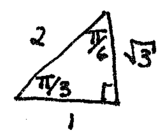
(a) $\int_1^4 \frac{(x-2)^2}{\sqrt{x}}\,dx = \int_1^4 \frac{x^2-4x+4}{\sqrt{x}}\,dx = \int_1^4 (x^{3/2} - 4x^{1/2} + 4x^{-1/2})\,dx$

$= \frac{2}{5}x^{5/2} - 4\cdot\frac{2}{3}x^{3/2} + 4\cdot 2x^{1/2}\Big]_1^4$

$= \left[\left(\frac{2}{5}(4)^{5/2} - \frac{8}{3}(4)^{3/2} + 8\sqrt{4}\right) - \left(\frac{2}{5} - \frac{8}{3} + 8\right)\right]$

$= \frac{64}{5} - \frac{64}{3} + 16 - \frac{2}{5} + \frac{8}{3} - 8 = \frac{62}{5} - \frac{56}{3} + 8 = \frac{186 - 280 + 120}{15} = \frac{26}{15}$

(b) $\int_{\sqrt{2}}^2 \frac{1}{x\sqrt{x^2-1}}\,dx = \operatorname{arcsec} x\Big]_{\sqrt{2}}^2$



$\sec\theta = 2$

$\cos\theta = \frac{1}{2}$

$= \operatorname{arcsec} 2 - \operatorname{arcsec}\sqrt{2}$

$= \frac{\pi}{3} - \frac{\pi}{4}$

$= \frac{4\pi - 3\pi}{12} = \frac{\pi}{12}$

(c) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec^2 x\,dx = \tan x\Big]_{-\frac{\pi}{4}}^{\pi/4}$

$= \tan\frac{\pi}{4} - \tan\left(-\frac{\pi}{4}\right)$

$= 1 - (-1)$

$= 2$

(d) $\int_{-1}^{1} \frac{3}{x^4}\,dx$ DNE since there is an infinite discontinuity at $x=0$

3. If $g(x) = \int_{x^3}^{e^x} t^2 \cos t \, dt$, find $g'(x)$. Show each step.

$$g'(x) = \frac{d}{dx} \int_{x^3}^{e^x} t^2 \cos t \, dt = \frac{d}{dx}\left[\int_{x^3}^{\pi} t^2 \cos t \, dt + \int_{\pi}^{e^x} t^2 \cos t \, dt \right]$$

$$= \frac{d}{dx}\left[- \int_{\pi}^{x^3} t^2 \cos t \, dt + \int_{\pi}^{e^x} t^2 \cos t \, dt \right]$$

$$= -\left((x^3)^2 \cos(x^3)\right)(3x^2) + (e^x)^2 \cos(e^x)(e^x)$$

$$= e^{3x}\cos(e^x) - 3x^8 \cos(x^3)$$

4. If $\int_0^3 f(x)dx = 4$, $\int_3^6 f(x)dx = 4$ and $\int_2^6 f(x)dx = 5$, find the value of $\int_0^2 f(x)dx$. (Be sure to show all your work and use the properties of the definite integral that we know one at a time.)

Since $\int_0^6 f(x)\,dx = \int_0^3 f(x)\,dx + \int_3^6 f(x)\,dx = 4 + 4 = 8$,

and $\int_0^6 f(x)\,dx = \int_0^2 f(x)\,dx + \int_2^6 f(x)\,dx$,

$8 = \int_0^2 f(x)\,dx + 5$ and so $\int_0^2 f(x)\,dx = 8 - 5 = 3$.

5. Evaluate the following. **Explain your work.**

(a) $\dfrac{d}{dx}\displaystyle\int_0^{\frac{\pi}{2}} \sin^3 x\, dx$ $= 0$

since the definite integral is a number when evaluated and the derivative of a number is zero

(b) $\displaystyle\int_0^{\frac{\pi}{4}} \left(\frac{d}{dx}\sin^3 x\right) dx$ $= \sin^3 x \Big]_0^{\pi/4} = \left[\sin\frac{\pi}{4}\right]^3 - \left[\sin 0\right]^3$

$= \left(\frac{\sqrt{2}}{2}\right)^3 - 0$

$= \frac{2\sqrt{2}}{8}$

$= \frac{\sqrt{2}}{4}$

(c) $\dfrac{d}{dx}\displaystyle\int_x^1 \sin^3 t\, dt$ $= \dfrac{d}{dx}\left[-\displaystyle\int_1^x \sin^3 t\, dt\right]$

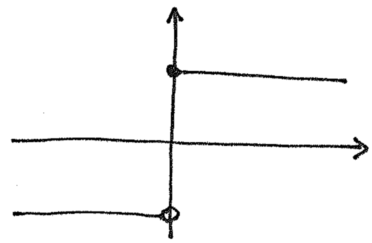
$= -\sin^3 x$

(d) $\displaystyle\int \left(\frac{d}{dx}\sin^3 x\right) dx$ $= \sin^3 x + C$ by the Fundamental Theorem of Calculus

6. Determine whether each of the following statements is true or false. If the statement is true, provide an appropriate justification. If the statement is false, provide a counterexample.

(a) If $f(x) \neq 0$ on $[a, b]$, then $\int_a^b f(x)dx \neq 0$.

FALSE. Consider $f(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}$



Then $\int_{-2}^{2} f(x)\,dx = -(2 \times 1) + (2 \times 1) = 0$

since we are finding the net area and the area beneath the x-axis is equal to the area above it, AND note that $f(x) \neq 0$ for all x.

(b) If f is continuous on an interval $[a, b]$ containing c, then $\int_a^b f(x)dx = \int_a^c f(x)dx - \int_b^c f(x)dx$.

TRUE!

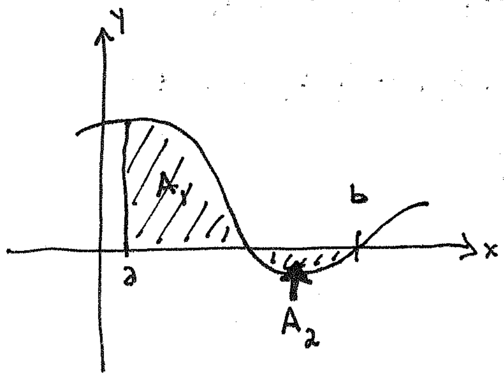
$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx = \int_a^c f(x)dx - \int_b^c f(x)dx$$

by properties of integration.

(c) If $\int_a^b f(x)dx \geq 0$, then $f(x) \geq 0$ for all x on $[a, b]$.

FALSE.

Consider the function $f(x)$ shown in the following graph:



Then $\int_a^b f(x)dx = A_1 - A_2 > 0$

since $A_1 > A_2$, but $f(x)$ is sometimes negative on $[a, b]$.

ANSWERS NOT SOLUTIONS FOR #7

7. Evaluate the following integrals. Be sure to show each step or explain your process. BE CAREFUL (that is, think carefully about your intervals)!

(a) $\int_0^{\pi} \sec^2 x \, dx$ DNE ... why?

(b) $\int_{-1}^{3} |2x - x^2| dx$ $= 4$ (Remember to express the integrand as a piecewise function.)