WEEK 4 LAB

MATH 131: Calculus II
February 13, 2020
Covering Sections 5.4-5.5

Your Name (Print): ANSWER KEY

1. Find the average value of $f(x) = |4 - x|$ on $[0, 6]$ **and** determine the points c where the average occurs. How do we know that such c must exist? Note that you can solve your definite integral in two ways. Be sure to show your work whichever way you choose.

$$f(x) = |4-x| = \begin{cases} 4-x & \text{if } x \le 4 \\ x-4 & \text{if } x > 4 \end{cases}$$

$$\bar{f} = \frac{1}{6-0}\int_0^6 |4-x|\,dx = \frac{1}{6}\left[\int_0^4 (4-x)\,dx + \int_4^6 (x-4)\,dx\right]$$

$$= \frac{1}{6}\left[\left[4x - \frac{x^2}{2}\right]_0^4 + \left[\frac{x^2}{2} - 4x\right]_4^6\right]$$

$$= \frac{1}{6}\left[\left(16 - \frac{16}{2}\right) - 0 + \left[\left(\frac{36}{2} - 24\right) - \left(\frac{16}{2} - 16\right)\right]\right]$$

$$= \frac{1}{6}\left[8 - 6 + 8\right] = \frac{1}{6}[10] = \frac{5}{3}$$

$|4-x| = \frac{5}{3}$ if $4 - x = \frac{5}{3}$ or $-(4-x) = \frac{5}{3}$

$\frac{4(3)-5}{3} = x$ or $x = \frac{5+4(3)}{3}$

$x = \frac{7}{3}$ or $x = \frac{17}{3}$ (both in interval!)

So the average occurs at $c = \frac{7}{3}$ and $c = \frac{17}{3}$.

We know that such a c exists by the Mean Value Theorem for Integrals.

2. Evaluate the following integrals. Be sure to show each step.

(a) $\displaystyle\int_{-1}^{0} (x-2)\sqrt{x+1}\,dx$

Let $u = x + 1$, then $du = dx$, and $x = u - 1$, so $x - 2 = u - 3$.

Change limits: $x = -1, \Rightarrow u = 0$; $x = 0, \Rightarrow u = 1$.

Therefore we have $\displaystyle\int_0^1 (u-3)u^{1/2}\,du = \int_0^1 u^{3/2} - 3u^{1/2}\,du = \frac{2}{5}u^{5/2} - 2u^{3/2}\Big|_0^1 = \frac{2}{5} - 2 = -\frac{8}{5}.$

(b) $\displaystyle\int_0^{1/4} \tan^3 \pi x \sec^2 \pi x \, dx$

Let $u = \tan(\pi x)$, then $du = \pi \sec^2(\pi x)\, dx$, so $\dfrac{1}{\pi} du = \sec^2(\pi x)\, dx$.

Change limits: $x = 0, \Rightarrow u = \tan(0) = 1$; $x = 1/4, \Rightarrow u = \tan(\pi/4) = 1$.

Therefore we have $\dfrac{1}{\pi}\displaystyle\int_0^1 u^3\, du = \dfrac{1}{4\pi} u^4\Big|_0^1 = \dfrac{1}{4\pi} - 0 = \dfrac{1}{4\pi}$.

(c) $\displaystyle\int_e^{e^2} \frac{1}{t \ln t}\, dt$

Let $u = \ln t$, then $du = \dfrac{1}{t} dt$.

Change limits: $t = e, \Rightarrow u = \ln e = 1$; $t = e^2, \Rightarrow u = \ln e^2 = 2$.

Therefore we have $\displaystyle\int_1^2 \frac{1}{u}\, du = \ln|u|\Big|_1^2 = \ln 2 - \ln 1 = \ln 2$.

(d) $\displaystyle\int \frac{t}{\sqrt{4 - 16t^4}}\, dt$

Note: $\displaystyle\int \frac{t}{\sqrt{4-16t^4}}\, dt = \int \frac{t}{\sqrt{4(1-4t^4)}}\, dt = \frac{1}{2}\int \frac{t}{\sqrt{1-4t^4}}\, dt$.

Let $u^2 = 4t^4$, so $u = 2t^2$, $du = 4t\, dt$, $\dfrac{1}{4} du = t\, dt$.

Therefore we have

$$\frac{1}{2}\int \frac{t}{\sqrt{1-4t^4}}\, dt = \frac{1}{8}\int \frac{1}{\sqrt{1-u^2}}\, du = \frac{1}{8}\arcsin u + C = \frac{1}{8}\arcsin 2t^2 + C$$

3. The phases of the moon occur in a regular, predictable 28 day lunar cycle. Data (for Boston) from The Old Farmer's Almanac indicate that the hours, $v(x)$, the moon is visible per day over the course of the cycle (assuming a cloudless sky) is $v(x) = 2.615\cos\left(\frac{\pi}{14}x - \pi\right) + 12.435$, where x is the day of the cycle. Find the average number of hours the moon is visible per day in Boston using calculus. Why does your answer make sense?

$$\bar{v} = \frac{1}{28-0}\int_0^{28}\left(2.615\cos\left(\frac{\pi}{14}x - \pi\right) + 12.435\right)dx$$

Final Answer: 12.435 hours per day on average

4. (a) Consider the following integral. For what value of n would the integral be easy? Fill in the value and solve: $\displaystyle\int \frac{t^n}{\sqrt{1-t^8}}\,dt$.

Try the value $n=7$.

(b) Consider the same integral. For what other value of n could you also solve the integral? Fill in that value and solve it: $\displaystyle\int \frac{t^n}{\sqrt{1-t^8}}\,dt$.

Try the value $n=3$.

Are there other values that work?

5. On the final exam for MATH 131, Elaine says that $\int xe^x\,dx = xe^x - e^x + C$. How can you check whether she is correct and should receive credit for her answer? Did she do it correctly?

Differentiate! $\frac{d}{dx}(xe^x - e^x + C) = xe^x + 1\cdot\cancel{e^x} - \cancel{e^x} + 0 = xe^x$ ✓

Yes! Elaine is correct!

6. (This was on the last lab, but I don't think anyone got to it in lab. Try it now!) Evaluate the following. **Explain your work.** You will need to use your knowledge of definite and indefinite integrals as well as the Fundamental Theorem of Calculus to do these problems.

(a) $\frac{d}{dx}\int_x^1 \sin^3 t\,dt$

☆ See Week 3 Lab! ☆

(b) $\int_0^{\frac{\pi}{4}} \left(\frac{d}{dx}\sin^3 x\right) dx$

(c) $\frac{d}{dx}\int_0^{\frac{\pi}{2}} \sin^3 x\,dx$

(d) $\int \left(\frac{d}{dx}\sin^3 x\right) dx$