WEEK 14 LAB KEY

MATH 131: Calculus II
December 5, 2019 Your Name (Print): ____________________

Covering Sections 8.3-9.2 and a review of some topics from the first half of the semester

This is a list of problems and it is not expected that enough room is provided for you to do all of them here. Start with Part 1; we will also put these on the board. It is not necessary to do the rest in order, choose on which section you would like to focus. **Please bring this to class tomorrow and Monday as well!**

Part 1: Absolute and Conditional Convergence

1. Determine whether the following converge absolutely, conditionally or not at all. Note that since we are interested in absolute convergence, we will start with the Absolute Convergence Test or the Ratio Test Extension. If we discover that the series is not absolutely convergent, then we can use the alternating series test to see if it is conditionally convergent.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(2n^4+7)}{6n^9-2n}$

This series is absolutely convergent. (Use the Absolute Convergence Test!)

(b) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt{n^2-1}}$

This series is conditionally convergent. (We did this one in lab together!)

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(n+1)!}{2^n n}$ (Hint: Try the Ratio Test!)

This series is divergent.

Part 2: Series

2. Determine whether the following series are convergent or divergent. If they are convergent, find the sum.

   (a) $\displaystyle\sum_{n=1}^{\infty}\left[2\left(\frac{3}{5}\right)^n+3\left(\frac{4}{9}\right)^n\right]$

   This series converges to $\dfrac{27}{5}$, using geometric series and the theorem about sums of series.

   (b) $\displaystyle\sum_{n=1}^{\infty}\frac{n}{\sqrt{n^2+4}}$

   This series is divergent by the Test for Divergence.

3. Determine whether the following series are convergent or divergent.

   (a) $\displaystyle\sum_{n=3}^{\infty}\frac{n^2}{e^n}$

   This series is convergent. (Try Ratio Test.)

   (b) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{\sqrt[3]{n(n+1)(n+2)}}$

   This series is divergent. (Try Limit Comparison Test.)

   (c) $\displaystyle\sum_{n=1}^{\infty}\frac{n\ln n}{(n+1)^3}$

   This series is convergent. (This is a tricky one! Try Comparison Test with the series $\displaystyle\sum_{n=1}^{\infty}\frac{\ln n}{n^2}$.)

4. Determine whether the following series converge absolutely, conditionally, or diverge.

   (a) $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n 2^{4n}}{(2n+1)!}$

   This series is absolutely convergent. (Try Ratio Test.)

   (b) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n-1}\frac{e^{\frac{1}{n}}}{n}$

   This series is conditionally convergent. (This means you will show that the Absolute Convergence Test fails, but the Alternating Series Test holds, as in 1(b).)

   (c) $\displaystyle\sum_{n=1}^{\infty}\frac{\cos(n\pi)(4n^3+1)}{n^4}$

   This series is conditionally convergent. (This means you will show that the Absolute Convergence Test fails, but the Alternating Series Test holds, as in 1(b).)

Part 3: True/False and Multiple Choice Questions

The final exam will have both True/False and Multiple Choice questions on it. When answering the True/False questions below, be sure to understand why your answer is correct and think about how you might reword the statement to have a different answer. Most of the Multiple Choice questions are regular problems. Work out your solution and find it among the choices given.

5. **True/False**

Circle T if the statement is **always** true or F if the statement is not always true.

T **F** $\int_a^b f(x)dx$ represents the area under the graph of f from $x = a$ to $x = b$.

T **F** If the sequence $\{a_n\}$ converges, then the sequence $\{(-1)^n a_n\}$ converges.

T F $\int_{-3}^{3} \frac{x^5}{\sqrt{12 - x^2}} dx = 0$

T **F** If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ is convergent.

***Can you explain carefully in a sentence why the false statements are false?)

6. **Multiple Choice**

Circle the number which best answers the question or completes the statement.

(A) If $g(x) = \int_{\sqrt{x}}^{2} \frac{t}{\sqrt{t^4 + 7}} dt$, then $g'(x) =$

(i) $\frac{\sqrt{x}}{\sqrt{x^2 + 7}}$

(ii) $-\frac{\sqrt{x}}{\sqrt{x^2 + 7}}$

(iii) $\frac{1}{2\sqrt{x^2 + 7}}$

(iv) $-\frac{1}{2\sqrt{x^2 + 7}}$

(v) none of the above

(B) For what values of c does the series $\sum_{n=1}^{\infty} (c - 1)^n$ converge?

(i) $-2 < c < 0$

(ii) $-2 < c \leq 0$

(iii) $-1 < c < 1$

(iv) $-1 < c \leq 1$

(v) $0 < c \leq 2$

(vi) $0 < c < 2$

Part 4: Revisiting some old friends

7. Solve the differential equation: $f''(x) = x + \sqrt{x}$, $f(1) = 1$, $f'(1) = 2$.

$$f(x) = \frac{x^3}{6} + \frac{4}{15}x^{\frac{5}{2}} + \frac{5}{6}x - \frac{4}{15}$$

8. Let $f(x) = 1 + 4x^2$ on the interval $[0, 3]$. Use six rectangles to estimate the area under the curve using right endpoints. Show a diagram with the rectangles. How accurate is your estimation?

I will let you draw the pretty picture! Then find $R_6 = \frac{1}{2}[f(\frac{1}{2}) + f(1) + f(\frac{3}{2}) + f(2) + f(\frac{5}{2}) + f(3)] = \cdots$. Finally, check your accuracy with an integral!

9. Use the **definition** of area to find the area under the graph $g(x) = 3x^2 + 7$ on the interval $[-1, 7]$. How can you check your answer?

$A = 400$; Remember after you use the definition with limits!, you can check your work using short-cuts!

10. Evaluate $\displaystyle\int_3^5 \frac{4}{x^2 - 4x + 3}dx$.

This is a divergent improper integral!

11. Sketch the region enclosed by $x + y^2 = 2$ and $x + y = 0$. Find the area of this region.

$$A = \frac{9}{2}$$

12. Sketch the region enclosed by $y = \cos x$ and $y = \cos^2 x$ between $x = 0$ and $x = \pi$. Find the area of this region.

$A = 2$

13. Sketch the region enclosed by $y = x^3$, $y = 2x + 4$ and $x = 0$.

(a) Find the volume of the solid generated by rotating the region about the x-axis.

$$V = \frac{1184\pi}{21}$$

(b) find the volume of the solid generated by rotating the region about the line $x = 3$.

$$V = \frac{512\pi}{15}$$

14. Find the arc length of $y = \frac{1}{2}x^2$ over the interval $[0, 4]$.

$$L = \frac{1}{2}[4\sqrt{17} + \ln(4 + \sqrt{17})]$$

15. Evaluate $\displaystyle\int x \arctan x \, dx$.

$\frac{x^2}{2} \arctan x - \frac{1}{2}x + \frac{1}{2} \arctan x + C$ Don't forget your family!

16. Suppose that 10 in-lb are needed to stretch a spring from its natural length of 20 inches to a length of 25 inches. How much work is needed to stretch it from 25 to 27 inches?

$W = \frac{48}{5}$ in-lbs

Part 5: Introducing some new friends: Power Series

17. Determine the radius and interval of convergence for the following power series:

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n}$

$R = 1$; $I = (-1, 1]$
Notes: use the Ratio Test to find the open interval and then plug in the endpoints to the series to determine if the series converges for those values.

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$

$R = \infty$; $I = (-\infty, \infty)$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{3n^2+1}$

$R = 1$; $I = [-1, 1]$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{4^n x^{2n}}{n+1}$

$R = \frac{1}{2}$; $I = [-\frac{1}{2}, \frac{1}{2})$