Guidelines for u-substitution

MATH 131: Calculus II

Suppose we are asked to evaluate the indefinite integral $\displaystyle\int f(x)\,dx$.

The Process

1. Try to rewrite the integrand in such a way that u-substitution is not necessary. If this is not possible or is lengthy (like multiplying out something raised to the fourth power), proceed to step 2.

2. Choose a u-substitution, say $u = g(x)$. (Be sure to write this step and the next down!)

3. Calculate $du = g'(x)\,dx$.

4. Try to use the equations in the previous two steps to transform the integrand into a form which involves **only the variable** u and which fits one of our original integration formulae. If you cannot eliminate x from the integrand or are left with a form that we do not already know how to integrate (see Section 4.9 on pages 318-327, especially Tables 4.9 and 4.10), return to step 2 and try a different u-substitution, otherwise proceed to step 5.

5. Evaluate the integral in step 4 to obtain a general antiderivative involving u.

6. Replace u in the antiderivative in step 5 by $g(x)$. Your final solution should contain only the variable x.

7. After all this, don't forget your family!

Choosing u

Don't be afraid to experiment when searching for your u. Below are some ideas for what to let u be. Often u will fulfill more than one of these criterion.

a) a function whose derivative (except for perhaps a constant) also appears in the integrand

b) a function that is the inner function of a composition of functions

c) a function that is raised to the highest power

d) a function that appears in the denominator

NOTE: If we are evaluating a definite integral, then we could use this same process and step 4 would include a rewriting of the limits of integration in terms of u. Also, step 6 would involve plugging in the limits of integration instead of going back to x, and step 7 would not be necessary since the result of a definite integral is a number, not a family of functions.