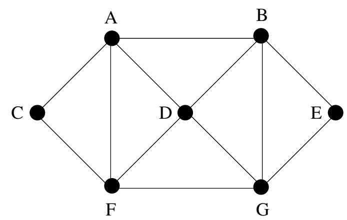
Groupwork: Graph Theory -
Eulerian and Hamiltonian properties

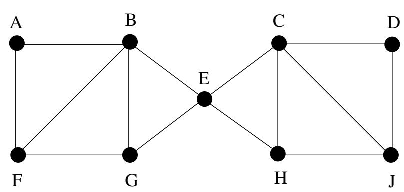
MATH 110: Discovering in Mathematics
November 12, 2019 Name (Print): ____________________

1. (i) Which of the following graphs are Eulerian (i.e. contain Euler circuits)? If the graph is Eulerian, write down an Eulerian circuit. Can you find more than one? If the graph is not Eulerian, explain why. (ii) Which graphs have Eulerian paths (this means that we can trace each edge exactly once but we do not end where we started)? If the graph has an Eulerian path, write it explicitly. Can you find more than one?

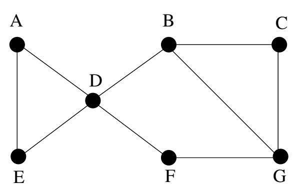
(a)



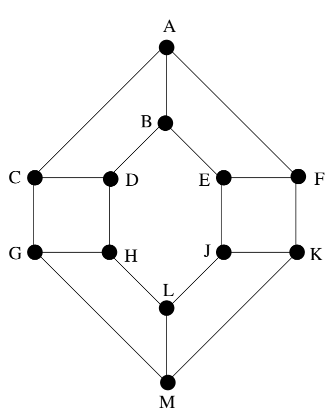
(b)



(c)



(d)



2. What property do the graphs in (1) that have Eulerian paths but are not Eulerian have?

3. For any of the graphs in (1) that do not have Eulerian paths, is it possible to alter the graph slightly to make it have one? Explain.

4. What if instead of trying to find a way to trace the graph so that we travel on each **edge** exactly once, we try to find a way to trace the graph so that we visit each **vertex** exactly once and end at the original vertex (so the only vertex we may repeat is the first when we arrive at it at the end of our travels)? This type of path is called a **Hamiltonian circuit**. In which graphs in (1) is it possible to find a Hamiltonian circuit? Justify.

5. For any of the graphs in (1) that do not contain a Hamiltonian circuit, is it possible to alter the graph slightly so that it does? Explain.

6. We said that if a graph has all vertices with even degree then we know it has an Eulerian circuit. Are there any conditions such that if we know a graph has that condition or conditions, then we know the graph has a Hamiltonian cycle? Experiment!

7. For each graph in (1), look at the sum of the degrees of the graph (that is, find the degree of each vertex and then add all of the degrees together). Compare this sum to other properties of the graph (especially other things you can count!). Make a conjecture about the sum of the degrees of the vertices of a graph. Justify/prove your conjecture.