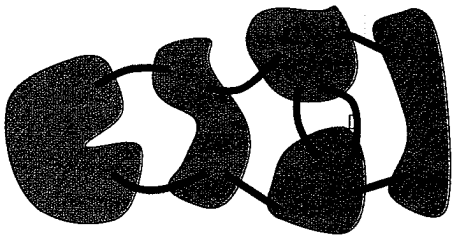
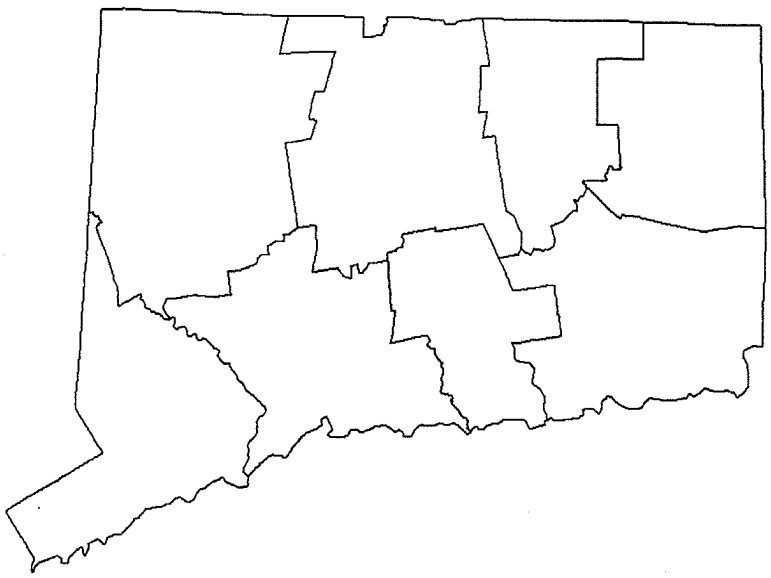
Mindscapes *Invitations to Further Thought*

In this section, Mindscapes marked **(H)** *have hints for solutions at the back of the book. Mindscapes marked* **(ExH)** *have expanded hints at the back of the book. Mindscapes marked* **(S)** *have solutions.*

I. Developing Ideas



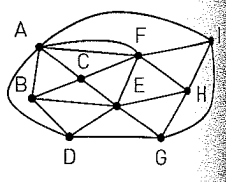
1. **Mapmaker, mapmaker, make me a graph.** Represent the map to the right using a graph, with a vertex (dot) for each landmass and an edge (line or arc) for each bridge.
2. **Unabridged list.** Represent each bridge from Mindscape I.1 using a pair of letters as in the text. Make a list of the bridges. Does each letter occur an even number of times in your list?
3. **Will the walk work?** Can you take a walk around the town shown in the map in Mindscape I.1, cross each bridge exactly once, and return to where you started? If yes, describe such a walk by listing the bridges. If not, explain.
4. **Connect Connecticut.** Below is a map showing the counties of Connecticut. Create a graph by placing one vertex (dot) in each county and then drawing an edge (line or arc) between two counties if they share a common boundary.



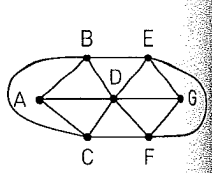
5. **Where's Waldoskova?** Do some research to find the location of Königsberg on a modern map. Does it still have the same name?

II. Solidifying Ideas

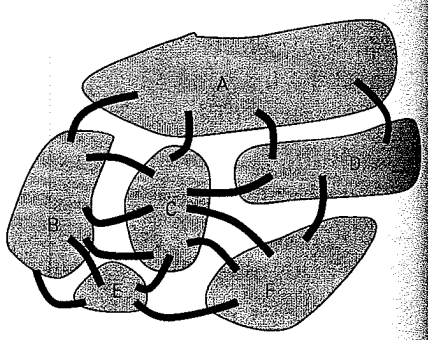
6. **Walk the line.** Is it possible to traverse the graph to the right with a path that uses each edge exactly once and returns to the vertex at which you started? If so, find such a path. If not, explain why not.



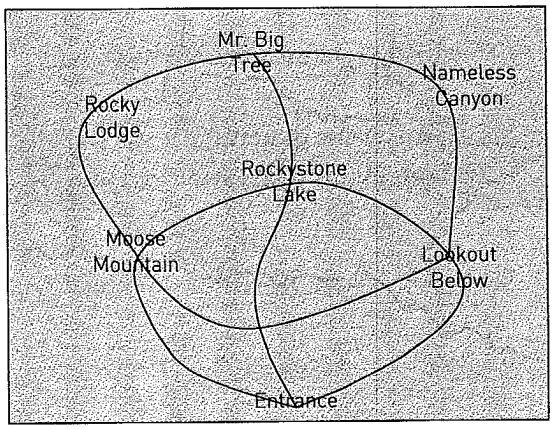
7. **Walkabout.** Is it possible to traverse the graph to the right with a path that uses each edge exactly once and returns to the vertex at which you started? If so, find such a path. If not, explain why not.



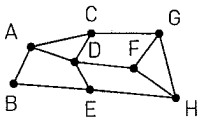
8. **Linking the loops.** In the map below, the following walks can be taken from various starting points: CAADDFFC, FCCBBCCEEF, DCCBBEE BBAAD. Can these walks be linked together to create one walk that starts on landmass C, crosses each bridge exactly once, and then returns to C?



9. **Scenic drive (S).** Below is a map of Rockystone National Park. One scenic drive is Entrance to Moose Mountain to Rockystone Lake to Lookout Below to Entrance. Can you add loop-d-loops to this drive to obtain a trip that traverses each road in the park exactly once and returns to the entrance?

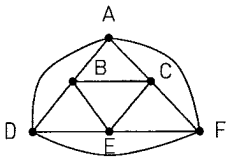


10. **Under-edged (H).** Is it possible to traverse the graph to the right with a path that uses each edge exactly once and returns to the vertex at which you started? If so, find such a path. If not, add the fewest number of new edges until such a path is possible. (Remember that edges can be curved.)

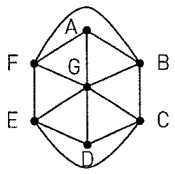


11. **No man is an island.** The country of Pelago consists of six islands. Use a pair of letters to represent each bridge in the map of Pelago to the right and make a list of the bridges. Does each letter occur an even number of times in your list? Can you create a path that traverses each bridge of Pelago exactly once and returns to the starting point? Why or why not?

12. **Path-o-rama.** For each graph below, determine if you can create a path that traverses each edge exactly once and returns to the starting vertex. If such a path is possible, present it. If not, explain why.



(a)



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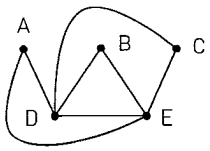
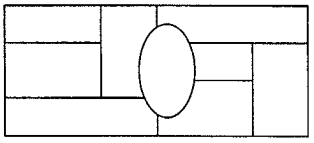


(c)

13. **Coloring Connecticut.** Return back to the map in Mindscape I.4. The Four Color Theorem asserts that it is possible to color the counties in this map using at most four colors so that counties that share a boundary get different colors. Is it possible to accomplish this with fewer than four colors? If so, describe such a coloring. If not, explain why not.

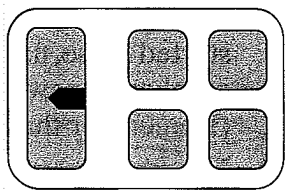
14. **Westward ho!** Here's a map of some states in the western United States. Create a graph with a vertex for each state and an edge between states that share a common boundary. The Four Color Theorem asserts that it is possible to color the states in this map using at most four colors so that states sharing a boundary get different colors. Is it possible to accomplish this with fewer than four colors? If so, describe such a coloring. If not, explain why not.

15. **A colorful museum.** The figure to the right shows the floor plan of a museum (doors and windows have been omitted). Create a graph with a vertex for each room and an edge between rooms that share a common wall. Is it possible to assign a paint color to each room so that rooms with shared walls get different colors? If so, describe such a coloring. If not explain.



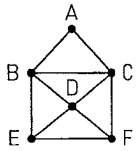
III. Creating New Ideas

16. **Snow job (ExH).** To the right is a map of the tiny town of Eulerville (the streets are white; the blocks are gray). After a winter storm, the village snowplow can clear a street with just one pass. Is it possible for the plow to start and end at the Town Hall (shown in black), clearing all the streets without traversing any street more than once?

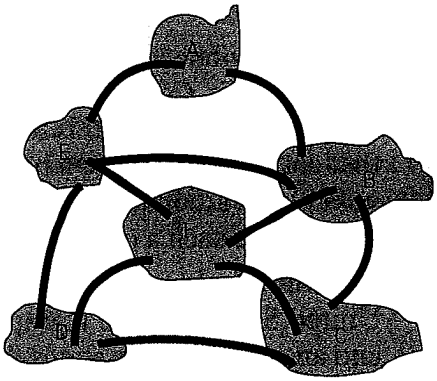


17. **Special delivery (ExH).** Julia is the letter carrier for the town of Eulerville shown in the previous Mindscape (the streets are white; the blocks are gray). The post office is located in the Town Hall (shown in black). Is there a route for Julia to use so she can start at the post office, deliver the mail along each street without having to retrace her steps, and return to the post office? (Assume that any street with a block on each side will have homes or business on both sides. Therefore Julia will have to walk along both sides of some streets to deliver the mail.)

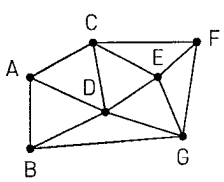
18. **Draw this old house.** Suppose you wanted to trace out the edges in the graph to the right without lifting your pen from the paper so that you draw each edge exactly once but with a new twist: You don't care if you return to the point where you started. Is it possible?



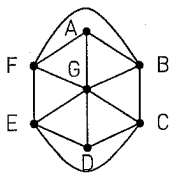
19. **Path of no return.** Consider the map to the right showing a town with six landmasses and 10 bridges. If you started on landmass C and walked as far as possible without going over the same bridge twice, where could you possibly get stuck? Why? What if you started on landmass D? What is special about C and D?



20. **Without a trace.** Is it possible to trace out either of the graphs below without crossing an edge more than once? (Don't worry about ending up where you started.) If yes, describe the path. If no, explain.



(a)



(b)