# Syllabus for MATH 135: First Steps into Advanced Mathematics

# Spring Semester 2020

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Class: held TTh: 2:00pm-3:30pm in Eaton 110 Textbook: *Discrete Mathematics with Ducks*, by sarah-marie belcastro Website: http://math.hws.edu/eking/FirstSteps/math135.html

# **Course Content and Organization**

First Steps into Advanced Mathematics is primarily a language course. We will be learning the precise language that produces rigorous proofs and reveals the beauty of mathematics. In order to have some basic material to which we can apply this language, we will also learn some discrete mathematics. Discrete mathematics is the study of mathematical objects that are not continuous. Think of the difference between graphing the line y = x and graphing the set of all points (x, x) where x is an integer. The former is a graph of a continuous function, while the second is not; it is discrete. The goal is for you to use the language (often within the realm of discrete mathematics) to begin to produce mathematics, not just consume it. We will develop our production skills by studying the Counting and Proofs, Sets and Logic, Graphs and Functions, Induction, and Fun with Cardinality chapters of the text, as well as studying a mathematical structure called posets with outside materials. Exploring these topics will increase our abilities to create, read, write, speak, and present mathematics.

This class will likely be very different from your previous mathematics courses partly because it will be run in a seminar style. There will be discussions, daily student presentations, and small group work. Very little time will be spent in lecture. You will be expected to read and work with material both **before** and after it is discussed in class. While previous courses may have challenged you to solve problems that were modifications of exercises seen before, here we begin to see that advanced mathematics requires more creativity. Here we begin to learn how to weave together sometimes seemingly unrelated facts in a logical way to create something new. Most students find both the format and the work for this class very challenging and engaging. I hope you will find the collaborative class sessions and office hours supportive and enlightening. While the course expectations may at first seem daunting, keep in mind that your classmates and I are embarking on this adventure with you. I think you will find the journey very rewarding (and even enjoyable!) as long as you stay involved.

# **Goals and Prerequisites**

The main goal of the course is for you to begin understanding how to discover and produce mathematics on your own. Here we also collect tools that will be used again in all upper level mathematics courses. These tools include the precise language of mathematics, many proof techniques, and some basic mathematical structures that will be invaluable throughout your mathematical career. By the end of the course you should know when to and how to apply the following proof techniques: direct, contrapositive, existence, uniqueness, contradiction, and induction. In addition, you should be familiar with the following mathematical structures and operations: basic logic (including implications, logical equivalence, and quantification), basic set theory and set operations, relations on sets (especially equivalence relations and partial orders), basic combinatorics (including product and sum principles, basic graph structures and properties, and congruence modulo n), and functions and some of their properties (injective, surjective, bijective). By the end of the term you should be confident in your ability to read and write several different types of proof. Your mathematical vocabulary will have increased particularly with respect to the language of sets, relations, and functions. You should feel more confident in presenting mathematics, both orally in class and in written assignments. In short, you should feel prepared to embark on the study of higher-level mathematics. This course significantly addresses Goal 3: the Quantitative Reasoning Goal.

In order to enroll for this class, you must have earned a C or better in MATH 131 (Calculus II) or earned AP credit for that class. This course is intended for students seriously considering becoming mathematics majors or minors. Since this is such a foundational course, I expect that this course will be top priority for you this semester. Indeed you should expect to spend at least 10 to 12 hours a week on this material outside of class; the more you do now, the stronger you will be for upper level mathematics courses. To reduce the amount of conscious time spent on this class, start assignments as soon as they are assigned! (You will be amazed at how much your mind can accomplish subconsciously if you give it the time!)

# **Textbook and Outline of Topics**

We will use our textbook regularly. You will be expected to read it regularly and we will use it for group work in class. At least one student from each group should bring a copy of the text to class each day (take turns where possible!). I have put a copy of the textbook on reserve in the library. In addition to the textbook, we will use a handout on posets which you will receive in April.

Weeks/Dates	Sections	Topics
Weeks 1–2	1.1 - 1.6	Counting and Proofs
Weeks 2–5	2.1 - 2.7	Sets and Logic
Tuesday, February 25	Exam 1	Chapters 1–2
Weeks 6–8	3.1 - 3.10	Graphs and Functions
Weeks 8–11	4.1 - 4.8	Induction
Tuesday, April 14	Exam 2	Chapters 3–4
Weeks 11-13	Handout	Posets
Week 14	15.1 - 15.5	Cardinality
Monday, May 11	Final Exam	Cumulative

This outline is meant to give you a general idea of how we will proceed through the text. It will be adjusted as necessary during the term.

# **Supplies**

In addition to the usual pencils, erasers, and paper, you will need an organized notebook (style of your choice: composition, three-ring binder, or spiral) to use as a journal. Your class notes may be kept in the same or in a different notebook. Please also obtain access to a stapler (This does not require buying one. For example, there is a stapler available for use in Eaton 113 for free.); collected work on more than one page should be stapled (not clipped) before submission.

#### Assessment

Reading and exercises will be assigned daily. Check the course website after each class for the assignment. These assignments will be divided into two types: journal assignments and collected assignments.

*Journal Assignments:* Journal assignments will be based on readings in the textbook and may include additional exercises not found in the text. Journal assignments should be completed before the beginning of the next class period.

Daily readings will usually only consist of a few pages, but should be carefully read several times. Please see the Strategies for Reading Mathematics handout! In your journal, keep a list of all new vocabulary (perhaps together at the back of your journal), and solve any assigned questions in or related to the reading. Also include at least one question you have about the material. Leave a space after each question so that you can fill in the answer when you discover it in class or office hours. Trying to figure out what exactly does not make sense to us is sometimes a difficult task, but a vital one as we search for clarity. Asking questions is an important skill for mathematicians to have as it leads to the ability to create interesting research questions. The Strategies for Reading Mathematics handout will give guidance in developing these questions. Be ready to ask your question(s) on the reading at the beginning of each class; it is your responsibility to be sure you find answers to your questions, and the first step is asking that it be addressed in class.

I hope that you find this book enjoyable to read. Note that the duck symbols guide you. There are portions of the text you should **not** read until you have worked on the previous problems. Be sure to observe the ducks with the stop signs warning this. The opportunity for you to work through and discover ideas on your own is invaluable; do not deny yourself that! Working through the reading assignments is the most important part of this course. You are encouraged to find a partner with whom you work some of this material. You should always come to class prepared to discuss and present on the reading. Although it is important that you eventually understand all of the material, it is not important for you to have all the right answers when we first discuss it. What is important is that you have taken time to carefully think and work through the ideas, and to identify specific questions. Many of the exercises you tackle in your journal work will be discussed in class. Take time to go back and look at your previous work and note if your proofs and solutions were correct and if not, why not. You may find a different approach to a problem than one shown in class. Remember: there is often more than one right way. If your way was not correct, try to use your mistakes to help you understand the material better. Even if your strategy did not work in one context, it may be helpful to illuminate a different concept, so do not automatically eliminate it! Your journal work gives us the opportunity to focus on the more complex questions in class, and helps to guide you to be a more independent mathematician.

Tuesday quizzes will be based on your journal work. Keep your journal organized so that you can find your responses and solutions for the quizzes! Here are suggestions for organizing your journal work:

- Keep exercises in the order they are assigned.
- Label each problem you are working on with its section, problem and page number. I will often only give you the section and problem number for the solution that you should copy from your journal onto the quiz.
- Use a highlighter to mark each label, so they are easily identifiable.

*Quizzes:* The majority of Tuesday classes will begin with a 10-15 minute open journal quiz. These quizzes will ask you to copy a problem from your journal, or give basic definitions and examples, share your question from the reading, or apply concepts studied in your journal work to new problems. Extra time will not be allowed for those arriving late to class. **Under no circumstances may a quiz be made up.** Your lowest quiz score will be dropped. Quizzes will be worth 20 points.

*Collected Homework:* Roughly once a week you will turn in a written assignment, usually on Fridays. These will ask you to demonstrate your ability to clearly and precisely express mathematical ideas in writing. This will include writing up proofs of theorems. Each proof will be assigned two grades – one grade for content and one grade for form. Content reflects your mastery of the mathematical concepts required for the proof and your use of appropriate proof methods. The grade for form will take into consideration clarity of expression, completeness, and proper usage of both English and mathematical grammar. Each homework set will be divided into two parts: Collaborative and Individual.

As the name suggests, Collaborative assignments may be discussed with your classmates. Although you may discuss the main ideas of the **Collaborative** assignments together (not Individual assignments as described below), details and your write-up must be entirely your own.

Individual assignments will consist of one or two questions, and must be done **individually** without help from any other people (that is, do not discuss it with other students, other faculty, etc.) or outside resources (that is, do not use the internet, other books, other students' notes, etc.) except your book, notes, and me. **Treat them like a take home exam.** Individual assignments also must be typed in  $IAT_EX$  (which we will discuss in class on Thursday), and must be submitted at least twice. The first time an Individual assignment is marked it will not receive a grade. Rarely is a proof complete in its first draft. Individual assignments may be resubmitted up to two times within one month of the original due date, but not after Monday, May 4th at 5pm. Details will be on the class website.

All assignments should be turned in via paper and not email unless otherwise directed. Your work should be done neatly in pencil or typed. Note that while assignments are due outside of class, **you are always welcome and encouraged to turn them in at the beginning of the class before they are due**. The point value of each written assignment will be determined by the length and complexity of the assignment, and thus will vary. Assignments will be considered late if they are not turned in by the time and date they are due. You may turn in two late assignments (any time before the Monday at 9am, 66 hours after they are due) without penalty. Beyond your two free lates, there will be a 10% penalty for each 24 hour period homework is late after the due date and time up to four days. Anything turned in four or more days late will have a 50% penalty. Be sure to turn in everything! Work will receive at least partial credit up until the time I return assignments to the class. Once I have returned Collaborative assignments to class and one month after Individual assignments are due, no credit can be earned, but work will be reviewed.

*Presentations:* Part of the class will consist of students presenting their journal and class work to each other. You will be expected to do your share in this. Much of the time I will rely on volunteers to make presentations. This makes it possible for you to present the work about which you feel most confident. But the fact that so much of your understanding depends on this participation means that you must volunteer on a regular basis. Don't assume that because others volunteer, you (or your grade) are off the hook. I may also assign presentations ahead of time or call on students randomly. Some presentations will be less formal, but formal presentations should be at least as well prepared as written work. Like written assignments, presentations will be given a grade for mathematical content and a grade for the form and quality of the presentation. Each student will likely have two formal presentations. However, this requirement may increase depending on our movement through the material. The formal presentation will be worth 20 points.

The person who is presenting their work at the board is not the only person with responsibilities in a presentation. The students sitting at their desks have as central a role to play. They should be making clarifying suggestions, asking questions and adding comments. If I think there is clarification needed or that there is confusion about the topic at hand, I will likely ask questions of those seated rather than of the presenter.

In addition to individual presentations, there will be at least one formal group presentation. Occasionally, some groups may be assigned to present and other groups may be assigned to evaluate a specific presentation. These will be worth 30 points.

Seminars: In addition to regular class time, you will be required to attend two mathematics/computer science seminar talks during the semester, at least one of which must be on mathematics. Seminars usually begin between 3pm and 5pm and last an hour (the days vary). Attendance at each talk is worth 10 points (you must be present and attentive for the entire talk to receive full credit).

*Bonus:* You may earn 5 bonus points for each additional mathematics/computer science seminar talk you attend. You may earn a maximum of 15 bonus points from seminar talks. Up to 10 points can be earned for each additional presentation. There is no maximum for bonus points earned through presentations.

Active participation: Your collected assignments, quizzes, presentations and seminar attendance will make up 47% of your grade.

*Exams:* There will be two midterm exams. The first will take place Tuesday, February 25th. Both exams will take place during our regular class time and optional review sessions will be scheduled the Sunday or Monday beforehand. Each midterm will be worth 15% of your course grade. The final exam will be Monday, May 11th from 8:30am until 11:30am. There may be a take-home or oral portion of the final. The final will be worth 23% of your course grade. It is impossible to construct fair makeup exams in mathematics. Thus, for your protection, my policy is that there are **no** makeup exams. **You must be present for all exams.** Any emergencies should be discussed with me and with your dean as soon as possible.

Attendance: Since your participation is such a large part of the class, absences will greatly affect your grade. More than two unexcused absences will lower your grade by at least one letter. The greater the number of absences, the greater the reduction. Excused absences require documentation such as a letter from a dean. Habitual tardiness will lower your grade. More than four absences will likely result in automatic failure. It is impolite to arrive late to class or leave the classroom while class is in session unless it is an emergency. Cell phones should be turned off and stowed during the entire class period.

# Disclaimer

The above exam dates, quantity of graded work, policies, and course layout are subject to change in the event of extenuating circumstances.

# The Center for Teaching and Learning (CTL)

At Hobart and William Smith Colleges, we encourage you to learn collaboratively and to seek the resources that will enable you to succeed. The Center for Teaching and Learning (CTL) is one of those resources: CTL programs and staff help you engage with your learning, accomplish the tasks before you, enhance your thinking and skills, and empower you to do your best. Resources at CTL are many: Study Mentors help you find your time and manage your responsibilities, Writing Fellows help you think well on paper, and professional staff help you assess academic needs, to name a few. I encourage you to explore these and other CTL resources designed to inspire your very best work. You can talk with me about these resources, visit the CTL office on the 2nd floor of the library to discuss options with the staff, or visit the CTL website at http://www.hws.edu/academics/ctl/index.aspx.

If you are a student with a "disability" (or what I like to call a "nontraditional approach to learning") for which you may need academic accommodations in this course, you should self-identify, provide appropriate documentation of your disability, and register for services with Disability Services at the Center for Teaching and Learning (CTL). Disability related accommodations and services will not be provided until the registration and documentation process is complete. The guidelines for documenting disabilities can be found at the following website: http://www.hws.edu/academics/ctl/disability\_services.aspx

Please direct questions about this process or Disability Services at HWS to Christen Davis, Coordinator of Disability Services, at CTL@hws.edu or x3351.

### Academic Integrity

I highly encourage you to form a small group with whom you can discuss some of the journal work and collaborative assignments. Verbalizing your questions, explaining your mathematical ideas and listening to others will increase your understanding. However, you should **not** feel free to copy someone else's work, ask someone else to edit your work, edit someone else's work, **or** make your work available to someone else. **Copying constitutes plagiarism, a violation of academic integrity which could result in failure in the course. There is, of course, no collaboration or use of outside resources (including other textbooks and the internet) allowed on individual assignments, quizzes, in-class or take-home exams (unless specifically noted).** Violation of the Colleges' Principle of Academic Integrity will likely result in a report sent to your file in the dean's office and/or appearance before the Committee on Standards.

# How to Succeed

- Attend all classes on time.
- Do the reading and journal assignments carefully. Use the Strategies for Reading Mathematics handout!
- Start assignments early so that you have time to work on the assignment several times and see me, if necessary. This will give your mind beneficial subconscious/unstructured time to solve problems on its own. Use the Strategies for Mathematics Problem Sets and Proofs handout.
- Spend some time each week working on the material from this class by yourself.
- Review the notes from the last class before coming to class again. Recopy the notes if appropriate. This makes preparation for a test easy.
- Participate in class discussions, and volunteer to present!
- Find a classmate with whom you can discuss the material outside of class.
- Ask me and your classmates lots of questions.

- Listen carefully to other students' ideas.
- Complete your write-ups on your own, including doing your own editing.
- Use my office hours liberally.
- Have fun!

# First Collected Homework

This assignment will contribute toward your active participation grade. It has two parts: a questionnaire and an essay. Go to the class website to print out the questionnaire. Fill it out and then submit it with your essay. Do not staple them together. For the essay, read the article: "The Secret to Raising Smart Kids", by Carol S. Dweck from *Scientific American*. The article can be found at this website: https://www.scientificamerican.com/article/the-secret-to-raising-smart-kids1/. Write a roughly one-and-a-half-to two page typed (10 or 12 point font and double-spaced) essay. This essay will be part reflection on the article and part autobiography. Discuss the following in your essay:

- where in your learning (this does not need to be restricted to the classroom) you have experienced struggle and how it was beneficial
- how you think the article applies to you or this course
- whether you are planning on being a mathematics major or minor
- why you chose to take this course
- your favorite and least favorite memories of mathematics
- your favorite mathematical topic
- what you expect to learn in this course
- your favorite hobbies, and anything else interesting (for example, what you did over winter break).

The paper is due by 4:00pm tomorrow (Wednesday, January 22nd). You may hand it in personally to me, place it in the homework collection box, or just slip it under my office door. This assignment also includes a short one-on-one meeting with me in my office after I have read your essay. Sign up for this appointment when you drop off your paper. Please bring a photo of yourself, with which you are willing to part, to the meeting (you do not need to have the photo when you turn in your essay). This meeting and the photo help me get to know each of you I do not, and catch up with those of you I do. Your grade on this assignment will be based on whether you address all the topics requested, as well as the quality of your writing (including good grammar and typography) and your prompt attendance at our meeting, photo in hand. It is critical that you are on time for your appointment! This assignment is worth 20 points. **Note:** if you had me for class in a previous semester you do not need to bring a photo.