Review for Exam 2

MATH 135: First Steps into Advanced Mathematics

Some Types of Problems:

Below are listed some of the types of problems that may appear on the exam. After most of them is listed a place to look for an example of such a problem. Note that there are variations of some of these types of problems that may not be covered by one example.

(1) determine whether or not a relation (or gipo) is a function, and then prove whether or not it is one-to-one or onto (Check Yourself problems at the end of Section 3.2)

(2) given a proof, correct it or evaluate it's accuracy

(3) prove or disprove statements dealing with basic definitions of graphs (such as cycles, degree sequences, etc.); Don't forget the First Theorem of Graph Theory (aka: the Handshaking Lemma)!

(4) determine whether two graphs are isomorphic (Section 3.8 Problems 2, 5 and 7)

(5) define what a Ramsey number is, know how to show $R(m, n)$ is NOT a particular value, and understand how to show that $K(3, 3) = 6$ (Section 3.9 Check Yourself problems 1 and 2)

(6) given the statement of a theorem, determine whether or not induction would be an appropriate way to prove it (Groupwork on March 21)

(7) prove a theorem by (regular or strong) induction (see sample questions on page two)

(8) make up a relation on a given set that has certain properties (Groupwork on March 23, problems 15-19)

(9) determine whether or not a relation is reflexive, symmetric, antisymmetric and transitive based on a given set of ordered pairs (Groupwork on March 23, problem 14)

(10) determine whether or not a relation is reflexive, symmetric, antisymmetric and transitive based on the definition of the relation (You Try It Problems 1.5 and 1.6)

(11) determine whether or not a relation is an equivalence relation or a partial ordering

Be sure to...

(1) review your definitions (such as tree, bijection, path, complete graph, etc.) and theorems.

(2) practice problems with**out** your book or notes.

(3) explicitly state which type of proof you are using.

(4) bring a pencil with a good eraser.

(5) ask me questions if you are stuck or need clarification.

(6) breathe!

NOTE: There may be true/false and short answer questions in addition to problems. For example, I may ask you to give an example of something (like a partial ordering with five elements on a given set, or a 3-regular graph on five vertices) or show there are no such examples. I could also ask you to explain the definition of a term.

NOTE: This is a **rough** guideline. The exam will be over Chapters 3 and 4 of *Discrete Mathematics with Ducks* and pages 1-5 of the Relations handout. You should be sure to review all of your homeworks, groupwork, journal and notes from these sections.

REMEMBER: Our exam will be Tuesday, April 11th from 7:15PM until 9:15PM in Napier 101.

REMEMBER: Bring your journal (a composition book) with you to the exam to be submitted. Catch up on any unfinished journal work and/or revise the work you have done as a review for the exam. Mark each entry clearly by exercise, problem or theorem number and highlight it. Make it clear which are rough drafts and which are final drafts if you entered an exercise more than once.

Induction Questions

Among other questions, Exam 2 will contain at least two theorems which you will be asked to prove by induction. Below are some examples of such theorems and you are highly encouraged to at least outline proofs for each of these.

1. For every $n \in \mathbb{N}$, $\displaystyle\sum_{j=1}^{n}(j \cdot j!) = (n+1)! - 1$.

2. If $x > 1$ is a given real number, then for every natural number $n \geq 2$, $(1+x)^n > 1 + nx$.

3. In a line of at least two students from the Colleges, if the first person is a William Smith student and the last person is a Hobart student, then somewhere in the line there is a Hobart student standing immediately behind a William Smith student. (Hint: Start by rewriting this statement so that it sounds more like induction!)

4. For all $n \in \mathbb{N}$, three divides $4^n - 1$.

5. For all $n \in \mathbb{N}$, the complete graph on n vertices, K_n has $\dfrac{n(n-1)}{2}$ edges. (Note that we are looking to prove this by induction now!)

6. Every polynomial is either irreducible or it is reducible and can be written as a product of irreducible polynomials. (Note: A polynomial is *reducible* if it can be written as the product of two polynomials of smaller degree. It is *irreducible* if it is not reducible.) (Hint: Start by rewriting this statement so it sounds more like induction. This can be done by doing induction on the degree of the polynomial.)

7. For every integer $n \geq 0$, $\displaystyle\sum_{i=0}^{n} 2^i = 2^{n+1} - 1$.