Groupwork: An Introduction to Graphs

MATH 313: Graph Theory
January 22, 2019 Name (Print): ____________________

You may need another piece of paper on which to complete some of this.

1. Consider the graph $G = (V, E)$ where $V = \{a, b, c\}$. For every possible size of G, give an example of a possible edge set for G both using set notation and using a diagram.

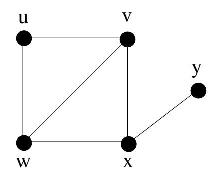
2. Draw representations of as many different graphs of order three as you can come up with.

3. Repeat question two for a graph of order four.

4. Given a graph G of order n, what is the smallest size possible for G? What is the largest? Use your work from questions 1-3 to come up with a conjecture. Can you justify your conjecture?

Definitions: A subgraph H of a graph G is said to be a **proper subgraph of** G if either $V(H)$ is a proper subset of $V(G)$ or $E(H)$ is a proper subset of $E(G)$. A subgraph H of a graph G is said to be a **spanning subgraph of** G if $V(H) = V(G)$. A subgraph H of a graph G is said to be an **induced subgraph of** G if (a) H is a subgraph of G and (b) whenever $u, v \in V(H)$ and u is adjacent to v in G, then u is adjacent to v in H.

5.



(a) Given the graph G above, draw a proper subgraph of G. Describe carefully in graph theoretic terms why it is proper.

(b) Given the graph G above, draw a spanning subgraph of G. Describe carefully in graph theoretic terms why it is spanning.

(c) Given the graph G above, draw subgraph of G that is NOT induced. Describe carefully in graph theoretic terms why it is not induced.

(d) Draw all the subgraphs of G. For each, determine whether the subgraph is proper, induced, or spanning.

6. Were any of your subgraphs in question five proper, induced AND spanning? If so, will there always be such a subgraph? If not, will there ever be? Why or why not?

7. Is it possible that you can have a subgraph H of a graph G such that H and G have different vertex sets, but they have the same edge set? Why or why not?

8. When someone asks me what I do, one of the examples that I like to use is the following: suppose you want to plow the snow from all the streets in your town and you want to do it in such a way that you never have to travel over the same street more than once. Is this possible? How do I connect this to graph theory? Describe how you would create a graph theoretical model of this question. This involves describing:

1. what vertices would represent,
2. what edges would represent (that is, creating a rule that describes when two vertices would be connected), and
3. rewording the problem in graph theoretic terms.

Then create a graph theoretic model using a portion of the map of part of Geneva on the next page (limit yourself to about ten vertices).