Hamiltonicity

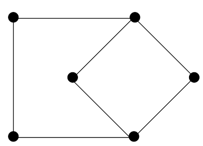
MATH 313: Graph Theory
Due April 15, 2016 at 3:00pm Name (Print): ______________________

This is part of the Collected Homework Assignment for this week.

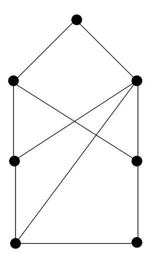
1. Showing step-by-step procedures (such as in Figure 6.19 at the top of page 149 in our text), find the closure of the following graphs.

(a) The star graph $K_{1,6}$

(b)



(c)



(d) The complete bipartite graph $K_{3,4}$

(e) The complete bipartite graph $K_{4,4}$

2. Using Theorem 6.9, determine which of the graphs in (1) are Hamiltonian. Is the Theorem helpful for these graphs? Do you think it is helpful in general? Why or why not?

3. In the text, there is a sufficient condition for Hamiltonicity on page 150 by Pósa that essentially uses degree sequences. Here is another sufficient condition that uses degree sequences:

> **Theorem (Chvátal, 1972)**: Let G be a graph with vertex degrees $d_1 \leq d_2 \leq \cdots \leq d_n$, where $n \geq 3$. If $i < \frac{n}{2}$ implies $d_i > i$ or $d_{n-i} \geq n - i$ for each i, then G is Hamiltonian.
>
> [Note: "$i < \frac{n}{2}$ implies $d_i > i$ or $d_{n-i} \geq n - i$ for each i" is referred to as Chvátal's condition.]

(a) The proof of this theorem essentially shows that graphs with these properties have closures isomorphic to complete graphs. How is this helpful?

(b) Show how this theorem applies to the following graph. Note that you will need to include the degree sequence and investigate whether or not the graph satisfies Chvátal's condition. This does not involve a lot of work, but you do need to show details.