Homework Week 7

MATH 204: Linear Algebra   Due October 12, 2018 **by 1:55pm**

   Remember that although you may discuss this assignment with others, your write up should be your own. **Do not share your write-up, look at other's write-ups, discuss word for word how something should be proved, etc.** Be sure to note with whom you collaborate if you do collaborate. Complete these exercises on a separate paper.

   Remember to distinguish clearly between vectors and scalars! You must make it clear to earn full credit!

1. Consider the matrices A defined in Exercises 4, 6 and 10 of Section 1.8 and do the following for each one; this means you have three questions with two parts each. (**Ignore the text instructions, and ignore the given vector b.**)

   (a) Decide whether the matrix transformation $T(\mathbf{x}) = A\mathbf{x}$ in question is onto its co-domain. Justify your answers with a short proof and appropriate theorems/facts.

   (b) Decide whether the matrix transformation $T(\mathbf{x}) = A\mathbf{x}$ in question is one-to-one. Justify your answers with a short proof and appropriate theorems/facts.

2. Rework question 5 from Exam 1, which is: "Suppose A is an $m \times n$ matrix with the property that for all $\mathbf{b}$ in $\mathbb{R}^m$ the equation $A\mathbf{x} = \mathbf{b}$ has at most one solution. **Use the definition of linear independence** to explain why the columns of A must be linearly independent. (Note you must use the definition of linear independence in your argument.)" [I will expect more precision on this as a homework question than I would as an exam question. Also, if you do not use the definition of linear independence for this question, you will not earn any points.]

3. Prove that if the columns of B are linearly dependent, then so are the columns of AB. Be sure to use appropriate theorems and facts.