Homework Week 10

MATH 204: Linear Algebra Due November 2, 2018 **by 1:55pm**

Remember that although you may discuss this assignment with others, your write up should be your own. **Do not share your write-up, look at other's write-ups, discuss word for word how something should be proved, etc.** Be sure to note with whom you collaborate if you do collaborate. Complete these exercises on a separate paper.

Remember to distinguish clearly between vectors and scalars! You must make it clear to earn full credit!

Pre-problem set work: Likely you used the result in Exercise 31 of Section 3.2 (page 178) on your last homework. Whether or not you did, be sure you know how to prove the result now. This is not part of your assignment to be turned in, but it is a result you should know and know how to prove.

1. An $n \times n$ invertible matrix U is **orthogonal** if $U^{-1} = U^T$. Prove that if U is orthogonal, then $\det(U) = \pm 1$.

2. This example will help you understand that vector spaces depend on both the set and how the operations of addition and scalar multiplication are defined. Let $\mathbb{R}^+ = \{x \in \mathbb{R} : x > 0\}$, the set of positive real numbers. Since the operations are not the usual ones, we will use the symbols discussed in class. Define "addition" as multiplication: $x \oplus y = xy$ and "scalar multiplication" as raising to a power: $c \odot x = x^c$. Using these operations write full proofs of the following:

(a) Verify Axiom 1 (Here your proof should begin with "Let $x, y \in \mathbb{R}^+$" ....).

(b) Verify Axiom 2.

(c) Verify Axiom 4.

(d) Verify Axiom 6.

(e) Verify Axiom 8 (Note that this looks like: $(c + d) \odot x = (c \odot x) \oplus (d \odot x)$.).

(I am only asking you to show me proofs that five of the axioms hold. You should check the rest as well to be sure you believe this is a vector space!)

3. Let H be the set of all singular 2×2 matrices. Is H a subspace of M_{22}? Carefully justify your answer with a proof or counterexample.