Homework Week 13

MATH 204: Linear Algebra Due November 30, 2018 **by 1:55pm**

Remember that although you may discuss this assignment with others, your write up should be your own. **Do not share your write-up, look at other's write-ups, discuss word for word how something should be proved, etc.** Be sure to note with whom you collaborate if you do collaborate. Complete these exercises on a separate paper.

Remember to distinguish clearly between vectors and scalars! You must make it clear to earn full credit!

1. Find a basis for the subspace of M_{22} spanned by the matrices

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -2 & 0 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 3 & -1 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} 5 & -3 \\ 3 & -4 \end{bmatrix}, \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}.$$

(Hint: Since we add and scalar multiply matrices just like we do with vectors in $\mathbb{R}^n$, we can think of matrices in M_{22} as vectors in $\mathbb{R}^4$ by listing all of the matrix entries vertically in a column. Now you can solve the problem in $\mathbb{R}^4$ and convert the answer back to matrices! Cool!)

2. Consider the linear transformation $T : \mathbb{R}^2 \to \mathbb{P}_2$ defined by $T\left(\begin{bmatrix} a \\ b \end{bmatrix}\right) = (a+b) + at + (b-a)t^2$. Show that T is one-to-one. (Hint: One method is to determine ker T. Why? After you find the kernel, try supposing $T(\vec{u}) = T(\vec{v})$.)

3. Let $\vec{p}_1(t) = 1 + t^2$, $\vec{p}_2(t) = 2 - t + 3t^2$, $\vec{p}_3(t) = 1 + t - 3t^2$.

   (a) Use coordinate vectors to show that these vectors are a basis for $\mathbb{P}_2$.

   (b) Consider the basis $\mathcal{B} = \{\vec{p}_1, \vec{p}_2, \vec{p}_3\}$ for $\mathbb{P}_2$. Find $\vec{q}$ in $\mathbb{P}_2$ given that $[\vec{q}]_{\mathcal{B}} = \begin{bmatrix} -3 \\ 1 \\ 2 \end{bmatrix}$.

   (c) If $\vec{r}(t) = 3 + t + 5t^2$, determine $[\vec{r}]_{\mathcal{B}}$.

4. Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation that is one-to-one. What is the dimension of the Range of T? (Hint: This linear transformation is a matrix transformation!) Justify with a proof. (Note that a definition from Section 4.5 is necessary for completing this question.)

5. Let $\mathcal{W}$ be the subspace spanned by the vectors $\begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 4 \\ 1 \end{bmatrix}$, $\begin{bmatrix} -8 \\ 6 \\ 5 \end{bmatrix}$, and $\begin{bmatrix} -3 \\ 0 \\ 7 \end{bmatrix}$. Find the dimension of $\mathcal{W}$. Justify your conclusion carefully.