

# Homework Week 11

MATH 204: Linear Algebra

Due November 9, 2018 by 1:55pm

Remember that although you may discuss this assignment with others, your write up should be your own. **Do not share your write-up, look at other's write-ups, discuss word for word how something should be proved, etc.** Be sure to note with whom you collaborate if you do collaborate. Complete these exercises on a separate paper.

Remember to distinguish clearly between vectors and scalars! You must make it clear to earn full credit!

1. Let  $K$  be the set of all  $2 \times 2$  matrices of the form  $\begin{bmatrix} a & b \\ c & 0 \end{bmatrix}$  where  $a + b = 0$ . Is  $K$  a subspace of  $M_{22}$ ? Carefully justify your answer with a proof or counterexample.

2. Let  $W = \left\{ \begin{bmatrix} 2s + 4t \\ 2s \\ 2s - 3t \\ 5t \end{bmatrix} \in \mathbb{R}^4 : s, t \in \mathbb{R} \right\}.$

- (a) Give an explicit example of a non-zero vector  $\vec{v} \in W$ .
- (b) Is  $W$  a subspace of  $\mathbb{R}^4$ ? Justify your answer very carefully with a proof (Hint: If it IS a subspace, feel free to use a very nice theorem!).

3. Let  $J = \left\{ \begin{bmatrix} 4a + 3b \\ a + 3b + c \\ 1 \\ 3b - 2c \end{bmatrix} \in \mathbb{R}^4 : a, b, c \in \mathbb{R} \right\}.$

- (a) Give an explicit example of a non-zero vector  $\vec{w} \in J$ .
- (b) Is  $J$  a subspace of  $\mathbb{R}^4$ ? Justify your answer very carefully with a proof (Hint: If it IS a subspace, feel free to use a very nice theorem!).

4. Assume  $A$  is an  $n \times n$  matrix. Prove: If  $\text{Nul } A = \{\vec{0}\}$ , then  $\det(A^T) \neq 0$ .

5. Let  $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 6 & 7 & 8 & 9 \end{bmatrix}$ . Give explicit descriptions of  $\text{Nul } A$  and  $\text{Col } A$ , i.e. as spans of sets of vectors.