Homework Week 14

MATH 204: Linear Algebra

Due December 10, 2018 by 1:55pm

Remember that although you may discuss this assignment with others, your write up should be your own. Do not share your write-up, look at other's write-ups, discuss word for word how something should be proved, etc. Be sure to note with whom you collaborate if you do collaborate. Complete these exercises on a separate paper.

Remember to distinguish clearly between vectors and scalars! You must make it clear to earn full credit!

- 1. If the null space of a 5×6 matrix A is 4-dimensional, what is the dimension of the row space of A? Clearly explain your answer defending your claim with definitions and theorems. (Note that clearly does not necessarily mean long!) Try solving this question using definitions, theorems and facts from chapter 4!
- 2. Suppose a nonhomogeneous system of seven linear equations in ten unknowns has a solution with three free variables. Is it possible to change some constants on the equations' right sides (i.e. to some coefficients in the solution vector) to make the new system inconsistent? Why or why not? Try solving this question using definitions, theorems and facts from chapter 4!
- 3. Suppose you are given a homogeneous system of nine linear equations in eleven variables. Is it possible that all solutions of the system are multiples of one fixed nonzero solution? Why or why not? Try solving this question using definitions, theorems and facts from chapter 4!
- 4. Find the eigenvalues and bases for the eigenspaces of $A = \begin{bmatrix} -1 & 0 & 1 \\ -3 & 4 & 1 \\ 0 & 0 & 2 \end{bmatrix}$.
- 5. Assume λ is an eigenvalue of A with corresponding eigenvector \vec{v} . Prove that λ is also an eigenvalue of A^T . (Hint: Try using properties of determinants, properties of transposes and the Characteristic Equation.)