Homework Week 3

MATH 204: Linear Algebra Due September 14, 2018 **by 1:55pm**

Remember that although you may discuss this assignment with others, your write up should be your own. **Do not share your write-up, look at other's write-ups, discuss word for word how something should be proved, etc.** Be sure to note with whom you collaborate if you do collaborate. Complete these exercises on a separate paper.

1. **Resource Management Application**: The NY State Fish and Game Department supplies three types of food to Seneca Lake that supports three species of fish. Each fish of Species 1 consumes, each week, an average of 1 unit of Food 1, 1 unit of Food 2, and 2 units of Food 3. Each fish of Species 2 consumes, each week, an average of 3 units of Food 1, 4 units of Food 2, and 5 units of Food 3. For a fish of Species 3, the average weekly consumption is 2 units of Food 1, 1 unit of Food 2, and 5 units of Food 3. Each week 15,000 units of Food 1, 10,000 units of Food 2, and 35,000 units of Food 3 are supplied to the lake. If we assume that all food is eaten, how many fish of each species can coexist in the lake? Is there a unique solution? Remember this is an application so that may influence how you describe your solution or solution(s). Carefully create a system of linear equations and use augmented matrices to solve this question.

2. Consider the set $H = \left\{ \begin{bmatrix} 4 \\ -4 \\ 2 \end{bmatrix}, \begin{bmatrix} -8 \\ 7 \\ -1 \end{bmatrix}, \begin{bmatrix} 8 \\ -6 \\ -2 \end{bmatrix} \right\}$. Is the vector $\begin{bmatrix} -32 \\ 4 \\ -7 \end{bmatrix}$ in Span(H)? If it is, write it as a specific linear combination of the vectors in H. If it is not, explain.

3. Consider the set $H = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} -6 \\ 7 \\ -5 \end{bmatrix} \right\}$. Is the vector $\begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix}$ in Span(H)? If it is, write it as a specific linear combination of the vectors in H. If it is not, explain.

4. **Curve Fitting Application**: Find the unique quadratic equation of the form $y = ax^2 + bx + c$ that goes through the points $(-2, 20)$, $(1, 5)$ and $(3, 5)$ in the xy-plane. (Hint: Create a system of equations with unknowns/variables a, b and c.) Isn't that cool!

5. Determine whether the vectors $\mathbf{v_1} = \begin{bmatrix} -1 \\ 3 \\ 5 \end{bmatrix}$, $\mathbf{v_2} = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$, $\mathbf{v_3} = \begin{bmatrix} 2 \\ 9 \\ 5 \end{bmatrix}$, span $\mathbb{R}^3$. Justify your conclusion with theorems and/or definitions.

6. Given $A = \begin{bmatrix} 1 & 2 & -1 \\ -3 & -4 & 2 \\ 5 & 2 & 3 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$, write the augmented matrix for the linear system that corresponds to the matrix equation $A\mathbf{x} = \mathbf{b}$. Then solve the system and write the solution as a vector. Lastly, check your work!

7. Ignore the instructions for this one! Instead, use the given A and $\mathbf{b}$ and describe the set of all $\mathbf{b}$ for which $A\mathbf{x} = \mathbf{b}$ has a solution. (Your description should be in the form of an equation involving b_1, b_2 and b_3.) Also, give a specific example of a $\mathbf{b}$ for which $A\mathbf{x} = \mathbf{b}$ does **not** have a solution, along with a few words of explanation. Number 16 from Section 1.4, page 41.