Homework Week 4

MATH 204: Linear Algebra | Due September 21, 2018 **by 1:55pm**

Remember that although you may discuss this assignment with others, your write up should be your own. **Do not share your write-up, look at other's write-ups, discuss word for word how something should be proved, etc.** Be sure to note with whom you collaborate if you do collaborate. Complete these exercises on a separate paper.

1. Let $A = \begin{bmatrix} 1 & 4 & 1 & 2 \\ 0 & 1 & 3 & -4 \\ 0 & 2 & 6 & 7 \\ 2 & 9 & 5 & -7 \end{bmatrix}$.

(a) Do the columns of A span $\mathbb{R}^4$? Justify your answer with at least one complete sentence.

(b) Does the equation $A\mathbf{x} = \mathbf{y}$ have a solution for each $\mathbf{y}$ in $\mathbb{R}^4$? Justify your answer with at least one complete sentence.

2. This question is designed to make you think about pivot positions in the rows and/or columns of a (coefficient) matrix A. The matrix and vectors in each part are not necessarily the same!

(a) Suppose A is a 6×6 matrix and $\mathbf{b} \in \mathbb{R}^6$ is a vector such that $A\mathbf{x} = \mathbf{b}$ has a unique solution. Let $\mathbf{c} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{bmatrix}$. Does the equation $A\mathbf{x} = \mathbf{c}$ have a solution? If so, is the solution unique? Prove your answer very clearly, justifying your assertions very carefully.

(b) Suppose A is a 5×6 matrix and $\mathbf{b} \in \mathbb{R}^6$. Is it possible that $A\mathbf{x} = \mathbf{b}$ has a unique solution? Carefully explain why or why not. Is it possible that the equation $A\mathbf{x} = \mathbf{c}$ has a solution for all $\mathbf{c} \in \mathbb{R}^6$? Prove your answer very clearly, justifying your assertions very carefully. Use an appropriate theorem.

3. Let $A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & 2 & 9 \\ 5 & 5 & h \end{bmatrix}$. For what values of h do the columns of A span $\mathbb{R}^3$? Be sure to show your work and justify your answer with an appropriate theorem.

Note this has a second page! Keep going!

4. Number 8 from Section 1.5, page 48. Be careful. You are given the coefficient matrix, not the augmented matrix for homogeneous system $A\mathbf{x} = \mathbf{0}$.

5. Here are some statement that require short proofs! Be sure to justify with definitions and show each step!

(a) Use the definition of scalar multiplication of vectors to prove: If $\mathbf{v} \in \mathbb{R}^n$, then $0\mathbf{v} = \mathbf{0_n}$, where the 0 on the left is a scalar 0 and $\mathbf{0_n}$ on the right is the zero vector in $\mathbb{R}^n$. (Note: This is usually written as $0\mathbf{v} = \mathbf{0}$, without the subscript.)

(b) Use part(a) and the DEFINITION of matrix-vector multiplication as a linear combination of the columns of A to prove: If A is an $m \times n$ matrix, then $A\mathbf{0_n} = \mathbf{0_m}$, where $\mathbf{0_n}$ on the left side of the equation is in $\mathbb{R}^n$ and $\mathbf{0_m}$ on the right side is in $\mathbb{R}^m$. (Note: This is usually written as $A\mathbf{0} = \mathbf{0}$ without the subscripts.)

6. Answer each of the following. Give clear, careful, short proofs that your answers are correct, using theorems and facts. Suppose A is a 5×5 matrix with 4 pivot positions.

(a) Must the equation $A\mathbf{x} = \mathbf{0}$ have a nontrivial solution?

(b) Must the equation $A\mathbf{x} = \mathbf{b}$ have a solution for EVERY $\mathbf{b} \in \mathbb{R}^5$?