Exam 2 Preparation

MATH 204: Linear Algebra

REMEMBER: Our exam will be Monday, October 22nd from 1:25PM until 2:50PM in Eaton 110. RECALL THE EARLY START TIME!!!

NOTE: Similar to the first exam, there will be short answer questions in addition to problems. For example, I could give you a few statements and ask you to determine whether each was true or false and to prove or give a counterexample for each. Similarly, I could ask you to give me an example of something or justify that no such example exists (our Possible/Impossible questions!).

NOTE: The exam will be over all the material covered in Sections 1.8, 1.9, 2.1-2.3, and 3.1. Note that although the *focus* of the exam will be on these sections, we still use the definitions and theorems from the previous sections to solve questions on this material. This is a **rough** guideline. You should be sure to review your homework, group work, quizzes and notes from these sections.

WARNINGS, Rules, Facts and Theorems: You should know and be able to use the following theorems and facts. Hopefully you already have these in your notes and/or on flashcards!

1. Linear Transformation Mapping of the Zero Vector (Fact on page 67)
2. Combining Vector Addition and Scalar Multiplication in Linear Transformations (Fact on page 67)
3. Standard Matrix for Linear Transformations Theorem (Theorem 1.10, page 72)
4. The Theorem on One-to-one Linear Transformations and Trivial Solutions (Theorem 1.11, page 77)
5. The Theorem on One-to-one and Onto and Properties of the Standard Matrix A (Theorem 1.12, page 78)
6. Basic Matrix Algebra for Scalar Multiplication and Addition (Theorem 2.1 page 95)
7. Product of Matrices and Linear Combinations of Columns Fact (Fact on page 97)
8. Basic Matrix Algebra for Multiplication of Matrices (Theorem 2.2, page 99)
9. Warnings About How Matrix Algebra Is Different from the Algebra of Real Numbers (Warnings on page 100)
10. Basic Matrix Algebra for the Transpose of Matrices (Theorem 2.3, page 101)
11. Invertibility of 2×2 Matrices Theorem (Theorem 2.4, page 105)
12. Invertibility and Unique Solutions Theorem (Theorem 2.5, page 106)
13. The Algebra of Inverses Theorem, Including the Shoes and Socks Theorem (Theorem 2.6, page 107)
14. Elementary Matrices and Row Operations on a Matrix A (Fact on page 108)
15. Invertibility of Elementary Matrices (Fact on page 109)
16. Invertibility and the Identity Matrix (Theorem 2.7, page 109)
17. The Invertible Matrix Theorem [Note: You should know this one by name!] (Theorem 2.8, page 114)
18. Invertibility and Square Matrices Fact (Fact on page 114)
19. Invertibility of Linear Transformations (Theorem 2.9, page 116)
20. Determinants and Cofactor Expansions Theorem (Theorem 3.1 page 168)
21. Determinants of Triangular Matrices (Theorem 3.2, page 169)

Definitions: You have been working hard on definitions! Be sure you have memorized these terms for the exam: linear transformation, domain, codomain, image, range, standard matrix, one-to-one transformation, onto transformation, transpose, invertible matrix, inverse, singular, non-singular, invertible transformation, elementary matrix, determinant, cofactor. You should know how to use these as well as have a good definition of them memorized.

Be sure to...

(1) remember to specify if you are right multiplying or left multiplying!

(2) remember to check if your matrix is square before applying the Invertible Matrix Theorem!

(3) review your definitions and theorems.

(4) practice finding examples that satisfy or do not satisfy particular requirements

(5) practice problems with**out** your book or notes or collaborators. (If you haven't done all the practice problems I assigned in these sections, go back and work through those. There are some really great questions!)

(6) bring a pencil (or several!) with a good eraser.

(7) ask me questions if you are stuck or need clarification.

(8) breathe!

Some Practice Exercises and Thoughts

1. Determine whether the columns of $A = \begin{bmatrix} 1 & 2 & 4 & 6 \\ 3 & 7 & -1 & 8 \\ 4 & 9 & 3 & 14 \\ 7 & -4 & 6 & 1 \end{bmatrix}$ are linearly independent. (Hint: You can use determinants!)
2. Prove that if A is an $n \times n$ invertible matrix, then A is row equivalent to A^T.
3. Prove that if A and B are square matrices and $AB = I$, then A and B are invertible and $B = A^{-1}$ and $A = B^{-1}$.
4. Suppose that you do not yet know the Invertible Matrix Theorem. How could you prove that if a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^n$ is not onto, then T is not one-to-one? (Note that T is a linear transformation with the same domain and range!)
5. What are four questions we might ask about a transformation T?
6. You should be able to determine whether an $n \times n$ matrix is invertible and if so, to find its inverse. HOW?