Exam 3 Preparation

MATH 204: Linear Algebra

REMEMBER: Our exam will be Friday, November 16, 2018 from 1:25PM until 2:50PM in Eaton 110. RECALL THE EARLY START TIME!!!

NOTE: Similar to the first two exams, there will be short answer questions in addition to problems. For example, I could give you a few statements and ask you to determine whether each was true or false and to prove or give a counterexample for each. Similarly, I could ask you to give me an example of something or justify that no such example exists.

NOTE: The exam will be over all the material covered in Sections 3.2 and 4.1-4.3. Note that although the *focus* of the exam will be on these sections, we still use the definitions and theorems from the previous sections to solve questions on this material. This is a **rough** guideline. You should be sure to review your homework, group work, quizzes and notes from these sections.

WARNINGS, Rules, Facts and Theorems: You should know and be able to use the following theorems and facts. Hopefully you already have these in your notes and/or on flashcards!

1. Row Operations and Determinants Theorem (Theorem 3.3, page 171)
2. Invertibility and Determinants Theorem (Theorem 3.4, page 173)
3. Determinant of the Transpose (Theorem 3.5, page 174)
4. Determinant of a Product of Matrices (Theorem 3.6, page 175)
5. Basic Properties of Vector Spaces (Facts, page 193)
6. Subspaces are Vector Spaces (Fact embedded in paragraph on page 195)
7. Spans are Subspaces (Theorem 4.1, page 196)
8. Nul A is a Subspace (Theorem 4.2, page 201)
9. Col A is a Subspace (Theorem 4.3, page 203)
10. Fact about when Col A spans $\mathbb{R}^m$ (Fact, page 204)
11. Characterization of Linear Dependence (Theorem 4.4, page 210)
12. The Spanning Set Theorem (Theorem 4.5, page 212)
13. Basis of Col A (Theorem 4.6, page 214)

Definitions: You have been working hard on definitions! Be sure you have memorized these terms for the exam: vector space, subspace, spanning set, null space, column space, linear transformation, kernel, range, onto, one-to-one, linear independence, linear dependence, linear dependence relation, basis and standard basis. You should know how to use these as well as have a good definition of them memorized.

Be sure to...

(1) review your definitions and theorems.

(2) practice finding examples that satisfy or do not satisfy particular requirements

(3) practice problems with**out** your book or notes or collaborators. (If you haven't done all the practice problems I assigned in these sections, go back and work through those. There are some really great questions!)

(4) bring a pencil (or several!) with a good eraser.

(5) ask me questions if you are stuck or need clarification.

(6) breathe!

Some Practice Exercises

1. Prove that if A is invertible, then $\det(A^{-1}) = \dfrac{1}{\det(A)}$. (Most of you used this fact. Some of you proved it already. All of you should know how to prove it! There are multiple ways that work!)

2. Suppose A and B are 3×3 matrices with $\det(A) = 3$ and $\det(B) = -2$. Justifying each step with a theorem, definition or fact, compute:

   - $\det(AB)$
   - $\det(B^{-1})$
   - $\det(A^3)$
   - $\det(B^T)$
   - $\det(A^{-1}BA)$
   - $\det(3B)$

3. For what values of k, if any, is $B = \left\{ \begin{bmatrix} k \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} k \\ 0 \\ 16 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ k \end{bmatrix} \right\}$ a basis for $\mathbb{R}^3$? Justify your answer!

4. Is $\mathcal{W} = \left\{ \begin{bmatrix} a \\ a^2 \end{bmatrix} : a \in \mathbb{R} \right\}$ a subspace of $\mathbb{R}^2$? Justify.

5. Is $\mathcal{W} = \left\{ \vec{p} \in \mathbb{P}_5 : \int_0^1 \vec{p}(t)dt = 1 \right\}$ a subspace of $\mathbb{P}_5$? Justify.

6. **(a)** Show that the transformation $T : \mathbb{R}^2 \to \mathbb{P}_2$ by $T\left(\begin{bmatrix} a \\ b \end{bmatrix} \right) = (a+b) + at + (b-a)t^2$ is linear.

   (b) Show that T is one-to-one. (Hint: One method is to determine $\ker T$. Why?)