Section 1.5: Solution Sets of Linear Systems

MATH 204: Linear Algebra
Prepare for class September 14, 2018   Name (Print): _____

After reading Section 1.5 (pages 43-47), answer the following questions.

1. Write down the definition of a **homogeneous** system of linear equations.

2. (a) Back in Section 1.1 we learned that a system of linear equations has three possibilities for how many solutions it may have. What were they?

    (b) Do these three possibilities still exist for a homogeneous system? Why or why not?

3. Define what a trivial solution is and what a nontrivial solution is.

4. Fill in the blank: FACT: The homogeneous equation $A\mathbf{x} = \mathbf{0}$ has a nontrivial solution if and only if

    _____.

5. Example: Determine if the following homogeneous system has a nontrivial solution.

$$2x_1 + 4x_2 + 6x_3 = 0$$
$$4x_1 + 5x_2 + 6x_3 = 0$$
$$3x_1 + x_2 - 2x_3 = 0$$

6. Example: Let $A = \begin{bmatrix} 1 & 0 & 6 \\ 1 & 1 & 2 \\ 3 & 2 & 10 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$.

(a) Describe the general solution for $A\mathbf{x} = \mathbf{b}$.

(b) Express the solution in parametric vector form (note outline for this on page 47).

(c) What is the solution in general parametric form for $A\mathbf{x} = \mathbf{0}$?

7. Write down the statement of Theorem 6 on page 47.

8. To prove Theorem 6, we actually need to show two statements. What are those two statements?

9. BONUS: Prove Theorem 6! That is, show that both statements you wrote in 8 are true!