

Vector Spaces

MATH 204: Linear Algebra
Prepare for class October 26, 2018

Name (Print): _____

After reading Section 4.1, work through the following ideas.

1. If I say that something is *closed* under an operation, what does that mean?

2. The definition of a vector space requires **ten** properties. Write down the definition of vector space. For each of its ten properties, write down the mathematical terminology for that property. For example, the third property is the *associative property of vector addition*.

3. On page 193 in the text, the authors list three facts. Write them here.

Note: There is a guide to prove these facts in Exercises 27-29 on page 199.

4. Example 4 on page 194 is an important example. Work through carefully (more carefully than the text!) through the example by filling in the details here...

Let $n \geq 0$ be an integer and let

\mathbb{P}_n = the set of polynomials of degree at most $n \geq 0$.

Members of \mathbb{P}_n have the form

$$\vec{p}(t) = a_0 + a_1t + a_2t^2 + \cdots + a_nt^n$$

where $a_0, a_1, a_2, \dots, a_n$ are real numbers and t is a real variable. We will prove that \mathbb{P}_n is a vector space.

Let $\vec{p}(t) = a_0 + a_1t + a_2t^2 + \cdots + a_nt^n$, $\vec{q}(t) = b_0 + b_1t + b_2t^2 + \cdots + b_nt^n$, and $\vec{r}(t) = c_0 + c_1t + c_2t^2 + \cdots + c_nt^n$ be elements of \mathbb{P}_n . Let c and d be scalars.

Prove Axiom 1 Holds:

The polynomial $\vec{p} + \vec{q}$ is defined as follows: $(\vec{p} + \vec{q})(t) = \vec{p}(t) + \vec{q}(t)$.

Therefore, $(\vec{p} + \vec{q})(t) =$ _____
 $=$ _____

which is also a _____ of degree at most _____.

So _____.

Prove Axiom 2 Holds:

Let \vec{p} and \vec{q} be in \mathbb{P}_n and as defined above.

Then $\vec{p} + \vec{q} =$ _____

while

$\vec{q} + \vec{p} =$ _____

Since addition of real numbers is _____, $a_i + b_i =$ _____.

So _____.

5. Write down any questions you have on the reading.