Vector Spaces and Subspaces

MATH 204: Linear Algebra
Prepare for class October 29, 2018 Name (Print): ______________________

After re-reading Section 4.1, work through the following ideas.

1. (The first part of this was on your last worksheet. Here it is all together!) Example 4 on page 194 is an important example. Work through carefully (more carefully than the text!) through the example by filling in the details here...

Let $n \geq 0$ be an integer and let

$$\mathbb{P}_n = \text{the set of polynomials of degree at most } n \geq 0.$$

Members of $\mathbb{P}_n$ have the form

$$\vec{p}(t) = a_0 + a_1 t + a_2 t^2 + \cdots + a_n t^n$$

where $a_0, a_1, a_2, \ldots, a_n$ are real numbers and t is a real variable. We will prove that $\mathbb{P}_n$ is a vector space.

Let $\vec{p}(t) = a_0 + a_1 t + a_2 t^2 + \cdots + a_n t^n$, $\vec{q}(t) = b_0 + b_1 t + b_2 t^2 + \cdots + b_n t^n$, and $\vec{r}(t) = c_0 + c_1 t + c_2 t^2 + \cdots + c_n t^n$ be elements of $\mathbb{P}_n$. Let c and d be scalars.

Prove Axiom 1 Holds:

The polynomial $\vec{p} + \vec{q}$ is defined as follows: $(\vec{p} + \vec{q})(t) = \vec{p}(t) + \vec{q}(t)$.

Therefore, $(\vec{p} + \vec{q})(t) =$ ____

$=$ ____

which is also a ____________________ of degree at most __________.

So ____.

Prove Axiom 2 Holds:

Let $\vec{p}$ and $\vec{q}$ be in $\mathbb{P}_n$ and as defined above.

Then $\vec{p} + \vec{q} =$ ____

while

$\vec{q} + \vec{p} =$ ____

Since addition of real numbers is ____________________, $a_i + b_i =$ ______________.

So ____.

Prove Axiom 3 Holds: (You provide all the details to this one...)

Prove Axiom 4 Holds:

The zero vector in $\mathbb{P}_n$ is defined as $\vec{0} = 0 + 0t + \cdots + 0t^n$.

Then $(\vec{p}+\vec{0})(t) = \vec{p}(t)+\vec{0} =$ ____

$=$ ____

So ____.

Prove Axiom 5 Holds:

If $\vec{p}$ is the polynomial in $\mathbb{P}_n$ as defined above then $-\vec{p} =$ ______________________________

because

$\vec{p} + (-\vec{p}) =$ ____

So ____.

Prove Axiom 6 Holds: (You provide all the details to this one...)

Prove Axiom 7 Holds: (You provide all the details to this one...)

Prove Axiom 8 Holds: (You provide all the details to this one...)

Prove Axiom 9 Holds: (You provide all the details to this one...)

Prove Axiom 10 Holds: (You provide all the details to this one...)

Thus ____.

2. What is the definition of subspace?

3. Give an example of a subspace **including of what it is a subspace**. You do not need to prove it, just write down what it is.

4. Give an example of something that is NOT a subspace **including of what it is NOT a subspace**. You do not need to prove it, just write down what it is.

5. State Theorem 4.1: The Spans are Subspaces Theorem.

6. Write down any questions you have on the reading.