Vector Spaces and Subspaces: Groupwork II

MATH 204: Linear Algebra
October 31, 2018 Name (Print): ____________________

Work together with your group to answer the following questions on Section 4.1.

1. This example will help you understand that vector spaces depend on both the set and how the operations of addition and scalar multiplication are defined. Let $V = \mathbb{R}^2$. Since the operations are not the usual ones, we will use the fancy symbols we discussed earlier. Define "addition" as: $(x, y) \oplus (w, z) = (x+w+1, y+z-2)$ and "scalar multiplication" as: $c \odot (x, y) = (cx + c - 1, cy - 2c + 2)$. Using these operations write full proofs of the following:

   (a) Verify Axiom 1 (Here your proof should begin with "Let $\vec{u}, \vec{v} \in \mathbb{R}^2$" ....).

   (b) Verify Axiom 2.

   (c) Verify Axiom 3.

   (d) Verify Axiom 4. (Careful! What is the additive identity here?)

   (e) Verify Axiom 5.

(I am only asking you to show me proofs that five of the axioms hold. You should check the rest as well to be sure you believe this is a vector space!)

For the following questions, let V be the vector space of the set of all functions from $\mathbb{R}$ to $\mathbb{R}$, and define addition and scalar multiplication as for functions. (You should at some point verify that you believe this is a vector space!!!)

2. Show that $\mathbb{P}_2$, the set of all polynomials of degree less than or equal to two, is a subspace of V.

3. Show that the set of all elements $\vec{f}$ in V satisfying $\vec{f}(1) = 4$ is NOT a subspace of V.

4. Show that the set of all elements $\vec{f}$ in V satisfying $\vec{f}(3) = 0$ IS a subspace of V.

5. Is the set of all elements $\vec{f}$ in V satisfying $\vec{f}(1) = \vec{f}(0)$ a subspace of V?