Groupwork: Linear Transformations, Kernal, Range and more!

MATH 204: Linear Algebra
November 7, 2018 Name (Print): _____

Work together with your group to answer the following questions relating to material in Section 4.2.

1. Recall that $\mathbb{P}_2$ is the set of all polynomials of degree less than or equal to two. That is, $\mathbb{P}_2 = \{\vec{p}(t) = a + bt + ct^2 : a, b, c \in \mathbb{R}\}$. Define a linear transformation $T : \mathbb{P}_2 \to \mathbb{R}^2$ by $T(\vec{p}) = \begin{bmatrix} \vec{p}(2) \\ \vec{p}(0) \end{bmatrix}$.

(a) Suppose $\vec{p}(t) = t^2 - t + 1$. What is the image of $\vec{p}$ under T?

(b) Find three vectors (polynomials) that span $\mathbb{P}_2$. (If you did your reading for Section 4.3, you should know another name for this set. What is it?)

(c) Show that T is a linear transformation.

(d) Determine the kernel of T. Write it explicitly as a spanning set.

(e) Determine the range of T.

(f) Is T one-to-one? Justify your answer clearly.

(g) Is T onto? Justify your answer clearly.