Groupwork: Null Spaces, Column Spaces and Linear Transformations

MATH 204: Linear Algebra
November 5, 2018 Name (Print): ______________________________

Work together with your group to answer the following questions on Section 4.2.

1. Here is a theorem that is a generalization of Theorem 4.3 in the text (do you see WHY it is a generalization?):

Theorem: Suppose V and W are vector spaces and $T : V \to W$ is a linear transformation. Then the range of T is a subspace of ______________________________.

(a) Fill in the blank above!

(b) Prove it! (Note: you cannot just refer to Theorem 4.3 since T is not necessarily a matrix transformation!)

2. Suppose $A = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$. Determine whether each of the following are in Nul A, Col A, both or neither. Justify your answers.

(a) $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

(b) $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

(c) $\begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix}$

(d) $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

(e) $\begin{bmatrix} 4 \\ 2 \end{bmatrix}$

3. Suppose $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 3 & 3 & 3 \end{bmatrix}$. Is Col $A = \mathbb{R}^3$? What is another way of wording this question?

4. Suppose $B = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$. Is Col $B = \mathbb{R}^3$?

5. Suppose $C = \begin{bmatrix} 1 & 2 & 0 & 0 & -1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$.

(a) Find an explicit description of the vectors in Nul C.

(b) What can you say about the vectors in the spanning set of Nul C (both here and in general)?

(c) What can you say about the columns of C (both here and in general)?

(d) If Nul C contains non-zero vectors, how many vectors are in the spanning set (both here and in general)?