

## Collected Homework Week 3

MATH 278: Number Theory  
Due February 1, 2015 at 4:00pm

Name (Print): \_\_\_\_\_

While you are welcome to discuss these problems with your classmates, your write-ups must be your own. Be sure to use your definitions and give details. Remember that your explanation is as important as the big ideas.

- [This is essentially Problem 3(c) from the first groupwork sheet.] Find the sum of the first two even natural numbers. Find the sum of the first three even natural numbers. Continue for several more sums and compare your answers. Can you find a pattern? Propose a conjecture and then prove it. (Note: this pattern is a little harder to see than the odd numbers, but it is there! Try finding different ways of rewriting the answers to each sum.)
- Choose a number  $x$  such that  $x > 20$ .
  - Choose another number  $y$  such that  $y > 50$  and  $y \equiv x \pmod{6}$ . Justify why your choice of  $y$  works.
  - Choose another number  $z$  such that  $z > 50$  and  $z \not\equiv x \pmod{6}$ . Justify why your choice of  $z$  works.
- Prove or disprove: If  $a|(b+c)$ , then either  $a|b$  or  $a|c$ .
- Prove:  $a|b$  if and only if  $ac|bc$ , where  $c \neq 0$ .