

Collected Homework Week 6

MATH 278: Number Theory
Due February 23, 2015 at 4:00pm

Name (Print): _____

1. Write a proof of Theorem 1.55 from page 22 of our text.
2. Write a clear, complete definition as requested in Exercise 1.56 from page 23 of our text. Think or look back at the way we made the definition of the greatest common divisor more precise than the version in the text! It is incredibly helpful to have a really good definition when we go to prove something! Also provide some examples as instructed in the exercise.
3. Write a proof of Theorem 1.57 from page 22 of our text.
4. Determine whether or not the following Diophantine equations can be solved. If not, explain why. If so, find all integer solutions. Use the Euclidean Algorithm where appropriate.
 - (a) $14x + 35y = 93$
 - (b) $24x + 138y = 18$
5. Find all solutions to the following puzzle posed by Alcuin of York in 775. Suppose 100 bushels of grain are distributed among 100 persons in such a way that each man receives 3 bushels, each woman 2 bushels, and each child $\frac{1}{2}$ bushel. How many men, women and children are there?

Notebook Problems Week 6

More Fun with Greatest Common Divisors

- (1) Prove that if $(a, b) = 1$, then $(a + b, a - b) = 1$ or 2. (Hint: Let $d = (a + b, a - b)$ and show that $d|2a$ and $d|2b$. Do you see how this can help?)
- (2) Prove that if $(a, b) = 1$, then $(a + b, ab) = 1$. (Hint: Let $d = (a + b, ab)$ and consider (a, d) .)