

Collected Homework Week 4

MATH 278: Number Theory
Due February 9, 2015 at 4:00pm

Name (Print): _____

While you are welcome to discuss these problems with your classmates, your write-ups must be your own. Be sure to use your definitions and give details. Remember that your explanation is as important as the big ideas.

1. Write a proof of Theorem 1.32 from page 18 of our text.
2. For each of the following, use the Euclidean Algorithm to find the GCD and then reverse the steps to express the GCD as a linear combination of a and b .
 - (a) $a = 889$ and $b = 229$
 - (b) $a = 357$ and $b = 629$
3. (a) Prove: Any perfect square will have a remainder of 1 or 0 when divided by 4. (We essentially discussed how to show this in class.)
 - (b) Given (a), what are the possible forms of a perfect square?
 - (c) Use the information you have found in the previous parts of this question to prove that $x^2 + 6x = 450$ has no integer solution.
 - (d) Experiment with what happens when you divide perfect squares by five. Make a conjecture about divisibility by 5 similar to the one in part (b) about divisibility by 4.
 - (e) Prove your conjecture from (d).