

Collected Homework Week 10

MATH 278: Number Theory
Due: April 1, 2015 at 4:00pm

Name (Print): _____

1. Prove the following statements:

(a) Show that $53^{103} + 103^{53}$ is divisible by 39.

(b) Show that 97 divides $2^{48} - 1$ using congruences. Then find some other prime factors of $2^{48} - 1$. (Feel free to use any results we have proved previously for the second part!)

(c) Prove that for all natural numbers n , 13 divides $3^{n+2} + 4^{2n+1}$.

2. (a) Develop a divisibility criteria statement similar to the statements in Corollaries 9 and 10 about divisibility by 11.

(b) A palindrome is a number which reads the same forward or backward like 17244271. Show that all palindromes with an even number of digits will be divisible by 11. (Isn't that cool!)

Notebook Problem Week 10

1. Find the remainder when $1^5 + 2^5 + 3^5 + 4^5 + \cdots + 99^5 + 100^5$ is divided by 4 using congruences, and, of course, prove your answer.