Collected Homework Week 11

MATH 278: Number Theory
Due: April 8, 2015 at 4:00pm

Name (Print): ______________________

1. Let's experiment a bit! Imagine you are a professional research mathematician and you are looking for a good statement to prove. Determine the residue of $(n-1)!$ modulo n for $n = 3$ through $n = 12$. From your results, come up with a conjecture. You do not need to prove your conjecture (for me...but perhaps for your profession!), but check to see if it works with two more examples.

2.

(a)Prove or disprove: $S = \{0, 1, 2, 2^2, 2^3, \ldots, 2^9\}$ forms a complete residue system modulo 11.

(b)Prove or disprove: $S = \{0, 1, 2, 2^2, 2^3, 2^4, 2^5\}$ forms a complete residue system modulo 7.

(c)Prove or disprove: $S = \{0, 1, 3, 3^2, 3^3, 3^4, 3^5\}$ forms a complete residue system modulo 7.

(d) Reflect on your work in parts (a)-(c). Briefly write about your reflections and make some observations and/or conjectures.

3. Prove that if $S = \{a_1, a_2, a_3, \ldots, a_n\}$ is a complete residue system modulo n, then for any integer c, $T = \{c + a_1, c + a_2, c + a_3, \ldots, c + a_n\}$ is also a complete residue system modulo n.

4. Find all solutions to $24x \equiv 12 \pmod{66}$. Be sure to show your work.

Notebook Problems Week 11

1. Prove that if $(a, n) = 1$, then for any integer b the set $S = \{b, b + a, b + 2a, b + 3a, \ldots, b + (n-1)a\}$ will form a complete residue system modulo n.

2. If p is prime and $a^2 \equiv 1 \pmod{p}$, prove that $a \equiv \pm 1 \pmod{p}$.