Practice Problems

(Final Answers)

1. Evaluate the following:

(a) $\lim_{\theta \to 0} \dfrac{\sin^2(4\theta)}{\theta^2}$ $= 16$

(b) $\lim_{\theta \to 0} \dfrac{\cos\theta - 1}{\sin\theta}$ $= 0$

2. Differentiate the following:

(a) $f(x) = \sec^2(13x^2) - \dfrac{8}{\sqrt[5]{x}}$ $\Rightarrow$

$$f'(x) = 52x\sec^2(13x^2)\tan(13x^2) + \frac{8}{5x^{6/5}}$$

(b) $g(x) = e^{\cos x}\tan(6x^3)$

$$g'(x) = e^{\cos x}\left[18x^2\sec^2(6x^3) - \sin x\tan(6x^3)\right]$$

(c) $h(x) = \dfrac{xe^x}{\sin x}$

$$h'(x) = \frac{e^x\left[x\sin x + \sin x - x\cos x\right]}{\sin^2 x}$$

3. Find the equations of the tangent lines to $f(x) = x\sqrt{5 - x^2}$ at $x = 1$ and $x = -2$.

$$y - 2 = \tfrac{3}{2}(x - 1)$$
$$y + 2 = -3(x + 2)$$

4. If $y = x^2\sin y$, find $\dfrac{dy}{dx}$.

$$\frac{dy}{dx} = \frac{2x\sin y}{1 - x^2\cos y}$$

5. Find $\dfrac{dy}{dx}$ if $3(x^2 + y^2)^2 = 100xy$.

$$\frac{dy}{dx} = \frac{25y - 3x^2 - 3xy^2}{3x^2y + 3y^3 - 25x}$$