Homework Assignment 11
Math 3001 — Fall 2014
Due Friday, November 31

Exercises:

5.2.1. Prove part (i) of Theorem 5.2.4 (part (ii) was done in class): Let f and g be functions defined on an interval A, and assume both are differentiable at some point $c \in A$. Then $(f+g)'(c) = f'(c) + g'(c)$.

5.2.6. (a) Assume that g is differentiable on $[a, b]$ and satisfies $g'(a) < 0 < g'(b)$. Show that there exists a point $x \in (a, b)$ where $g(a) > g(x)$, and a point $y \in (a, b)$ where $g(y) < g(b)$.

(b) Complete the proof of Darboux's Theorem started on page 136.

5.3.1. Recall from Exercise 4.4.9 that a function $f\colon A \to \mathbf{R}$ is Lipschitz on A if there exists an $M > 0$ such that

$$\left| \frac{f(x) - f(y)}{x - y} \right| \leq M$$

for all $x, y \in A$. Show that if f is differentiable on a closed interval $[a, b]$ and f' is continuous on $[a, b]$, then f is Lipschitz on $[a, b]$.

5.3.3. Let h be a differentiable function defined on the interval $[0, 3]$, and assume that $h(0) = 1$, $h(1) = 2$, and $h(3) = 2$.

(a) Argue that there exists a point $d \in [0, 3]$ where $h(d) = d$.

(b) Argue that at some point c we have $h'(c) = 1/3$.

(c) Argue that $h'(x) = 1/4$ at some point in the domain.

5.3.5. A *fixed point* of a function f is a value x where $f(x) = x$. Show that if f is differentiable on an interval with $f'(x) \neq 1$, then f can have at most one fixed point.