Prove the following.

Theorem

Let A, B, and C be sets where $B = \overline{A}$ and $C = \overline{B}$. Then $A = C$.

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Let A, B, and C be sets where $B = \overline{A}$ and $C = \overline{B}$. Then $A = C$.

That is, we are proving that $A = \overline{\overline{A}}$.
Why is that?

$$C = \overline{B} = \overline{\overline{A}}$$

(replace B with $\overline{A}$ in the last equality)
Proving $A = C$ is the same as showing $A = \overline{\overline{A}}$.

Theorem

Let $A, B,$ and C be sets where $B = \overline{A}$ and $C = \overline{B}$. Then $A = C$.

Definition (Set equality)

Let A and B be sets. Then $A = B$ if $x \in A \Rightarrow x \in B$ and $x \in B \Rightarrow x \in A$.

Given $A = B$, we know

◦	$x \in A$	$\Rightarrow$	$x \in B$	◦
◦	$x \in B$	$\Rightarrow$	$x \in A$	◦
◦	$x \notin A$	$\Rightarrow$	$x \notin B$	◦
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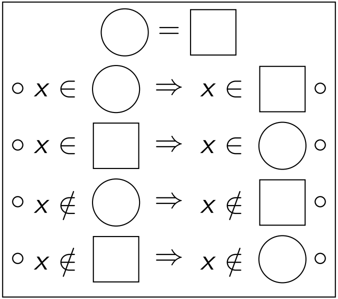

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Let A and B be sets. Then $A = B$ if $x \in A \Rightarrow x \in B$ and $x \in B \Rightarrow x \in A$.

Given $B = \overline{A}$, we know

∘	$x \in B$	$\Rightarrow$	$x \in \overline{A}$	∘
∘	$x \in \overline{A}$	$\Rightarrow$	$x \in B$	∘
∘	$x \notin B$	$\Rightarrow$	$x \notin \overline{A}$	∘
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Definition (Set equality)

Let A and B be sets. Then $A = B$ if $x \in A \Rightarrow x \in B$ and $x \in B \Rightarrow x \in A$.

Given $C = \overline{B}$, we know

- $x \in C \Rightarrow x \in \overline{B}$
- $x \in \overline{B} \Rightarrow x \in C$
- $x \notin C \Rightarrow x \notin \overline{B}$
- $x \notin \overline{B} \Rightarrow x \notin C$

Theorem

Let $A, B,$ and C be sets where $B = \overline{A}$ and $C = \overline{B}$. Then $A = C$.

Definition (Set complement)

Let A be a set. Then $\overline{A} = \{x : x \notin A\}$.

Given a set A, we know

∘	$x \in A$	$\Rightarrow$	$x \notin \overline{A}$	∘
∘	$x \notin \overline{A}$	$\Rightarrow$	$x \in A$	∘
∘	$x \notin A$	$\Rightarrow$	$x \in \overline{A}$	∘
∘	$x \in \overline{A}$	$\Rightarrow$	$x \notin A$	∘

Theorem

Let A, B, and C be sets where $B = \overline{A}$ and $C = \overline{B}$. Then $A = C$.

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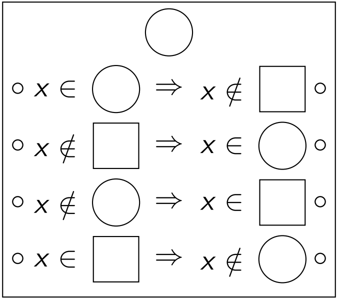

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∘	$x \in A$	$\Rightarrow$	$x \notin \overline{A}$	∘
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Let A, B, and C be sets where $B = \overline{A}$ and $C = \overline{B}$. Then $A = C$.

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∘	$x \notin \overline{B}$	$\Rightarrow$	$x \in B$	∘
∘	$x \notin B$	$\Rightarrow$	$x \in \overline{B}$	∘
∘	$x \in \overline{B}$	$\Rightarrow$	$x \notin B$	∘

Theorem

Let A, B, and C be sets where $B = \overline{A}$ and $C = \overline{B}$. Then $A = C$.

Proof.

We use a double inclusion argument to prove that $A = C$. That is, we will show that $A \subseteq C$ and $C \subseteq A$.

...

□

Theorem (Double inclusion principle)

Let A and B be sets. Then $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.

Theorem

Let A, B, and C be sets where $B = \overline{A}$ and $C = \overline{B}$. Then $A = C$.

Proof.

We use a double inclusion argument to prove that $A = C$. That is, we will show that $A \subseteq C$ and $C \subseteq A$.

($C \subseteq A$): In order to show that $C \subseteq A$, we show that every element of C is an element of A. Namely, $x \in C \Rightarrow x \in A$.

$\ldots$ □

Definition (Subset)

Let A and B be sets. Then $A \subseteq B$ if $x \in A \Rightarrow x \in B$.

Given $B = \overline{A}$, we know
∘ $x \in B \Rightarrow x \in \overline{A}$ ∘
∘ $x \in \overline{A} \Rightarrow x \in B$ ∘
∘ $x \notin B \Rightarrow x \notin \overline{A}$ ∘
∘ $x \notin \overline{A} \Rightarrow x \notin B$ ∘

Given $C = \overline{B}$, we know
∘ $x \in C \Rightarrow x \in \overline{B}$ ∘
∘ $x \in \overline{B} \Rightarrow x \in C$ ∘
∘ $x \notin C \Rightarrow x \notin \overline{B}$ ∘
∘ $x \notin \overline{B} \Rightarrow x \notin C$ ∘

Given the set A, we know
∘ $x \in A \Rightarrow x \notin \overline{A}$ ∘
∘ $x \notin \overline{A} \Rightarrow x \in A$ ∘
∘ $x \notin A \Rightarrow x \in \overline{A}$ ∘
∘ $x \in \overline{A} \Rightarrow x \notin A$ ∘

Given the set B, we know
∘ $x \in B \Rightarrow x \notin \overline{B}$ ∘
∘ $x \notin \overline{B} \Rightarrow x \in B$ ∘
∘ $x \notin B \Rightarrow x \in \overline{B}$ ∘
∘ $x \in \overline{B} \Rightarrow x \notin B$ ∘

$$x \in C \Rightarrow$$

$$\vdots$$

$$\Rightarrow x \in A$$

Given $B = \overline{A}$, we know		
$x \in B$	$\Rightarrow$	$x \in \overline{A}$
$x \in \overline{A}$	$\Rightarrow$	$x \in B$
$x \notin B$	$\Rightarrow$	$x \notin \overline{A}$
$x \notin \overline{A}$	$\Rightarrow$	$x \notin B$

Given $C = \overline{B}$, we know		
$x \in C$	$\Rightarrow$	$x \in \overline{B}$
$x \in \overline{B}$	$\Rightarrow$	$x \in C$
$x \notin C$	$\Rightarrow$	$x \notin \overline{B}$
$x \notin \overline{B}$	$\Rightarrow$	$x \notin C$

Given the set A, we know		
$x \in A$	$\Rightarrow$	$x \notin \overline{A}$
$x \notin \overline{A}$	$\Rightarrow$	$x \in A$
$x \notin A$	$\Rightarrow$	$x \in \overline{A}$
$x \in \overline{A}$	$\Rightarrow$	$x \notin A$

Given the set B, we know		
$x \in B$	$\Rightarrow$	$x \notin \overline{B}$
$x \notin \overline{B}$	$\Rightarrow$	$x \in B$
$x \notin B$	$\Rightarrow$	$x \in \overline{B}$
$x \in \overline{B}$	$\Rightarrow$	$x \notin B$

$$x \in C \Rightarrow x \in \overline{B} \qquad \text{(since } C = \overline{B}\text{)}$$

$$\vdots$$

$$\Rightarrow x \in A$$

Given $B = \overline{A}$, we know		
$x \in B$	$\Rightarrow$	$x \in \overline{A}$
$x \in \overline{A}$	$\Rightarrow$	$x \in B$
$x \notin B$	$\Rightarrow$	$x \notin \overline{A}$
$x \notin \overline{A}$	$\Rightarrow$	$x \notin B$

Given $C = \overline{B}$, we know		
$x \in C$	$\Rightarrow$	$x \in \overline{B}$
$x \in \overline{B}$	$\Rightarrow$	$x \in C$
$x \notin C$	$\Rightarrow$	$x \notin \overline{B}$
$x \notin \overline{B}$	$\Rightarrow$	$x \notin C$

Given the set A, we know		
$x \in A$	$\Rightarrow$	$x \notin \overline{A}$
$x \notin \overline{A}$	$\Rightarrow$	$x \in A$
$x \notin A$	$\Rightarrow$	$x \in \overline{A}$
$x \in \overline{A}$	$\Rightarrow$	$x \notin A$

Given the set B, we know		
$x \in B$	$\Rightarrow$	$x \notin \overline{B}$
$x \notin \overline{B}$	$\Rightarrow$	$x \in B$
$x \notin B$	$\Rightarrow$	$x \in \overline{B}$
$x \in \overline{B}$	$\Rightarrow$	$x \notin B$

$$
\begin{aligned}
x \in C &\Rightarrow x \in \overline{B} && \text{(since } C = \overline{B}\text{)} \\
&\Rightarrow x \notin B && \text{(by def. of converse of } B\text{)} \\
&\;\;\vdots \\
&\Rightarrow x \in A
\end{aligned}
$$

Given $B = \overline{A}$, we know		
$x \in B$	$\Rightarrow$	$x \in \overline{A}$
$x \in \overline{A}$	$\Rightarrow$	$x \in B$
$x \notin B$	$\Rightarrow$	$x \notin \overline{A}$
$x \notin \overline{A}$	$\Rightarrow$	$x \notin B$

Given $C = \overline{B}$, we know		
$x \in C$	$\Rightarrow$	$x \in \overline{B}$
$x \in \overline{B}$	$\Rightarrow$	$x \in C$
$x \notin C$	$\Rightarrow$	$x \notin \overline{B}$
$x \notin \overline{B}$	$\Rightarrow$	$x \notin C$

Given the set A, we know		
$x \in A$	$\Rightarrow$	$x \notin \overline{A}$
$x \notin \overline{A}$	$\Rightarrow$	$x \in A$
$x \notin A$	$\Rightarrow$	$x \in \overline{A}$
$x \in \overline{A}$	$\Rightarrow$	$x \notin A$

Given the set B, we know		
$x \in B$	$\Rightarrow$	$x \notin \overline{B}$
$x \notin \overline{B}$	$\Rightarrow$	$x \in B$
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\begin{aligned}
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\Rightarrow x &\notin B && \text{(by def. of converse of } B\text{)} \\
\Rightarrow x &\notin \overline{A} && \text{(since } B = \overline{A}\text{)} \\
&\vdots \\
\Rightarrow x &\in A
\end{aligned}
$$

Given $B = \overline{A}$, we know

$x \in B$	$\Rightarrow$	$x \in \overline{A}$
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$x \notin \overline{B}$	$\Rightarrow$	$x \notin C$

Given the set A, we know

$x \in A$	$\Rightarrow$	$x \notin \overline{A}$
$x \notin \overline{A}$	$\Rightarrow$	$x \in A$
$x \notin A$	$\Rightarrow$	$x \in \overline{A}$
$x \in \overline{A}$	$\Rightarrow$	$x \notin A$

Given the set B, we know

$x \in B$	$\Rightarrow$	$x \notin \overline{B}$
$x \notin \overline{B}$	$\Rightarrow$	$x \in B$
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$x \in \overline{B}$	$\Rightarrow$	$x \notin B$

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\begin{aligned}
x \in C \Rightarrow x \in \overline{B} & \qquad \text{(since } C = \overline{B}\text{)} \\
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\Rightarrow x \notin \overline{A} & \qquad \text{(since } B = \overline{A}\text{)} \\
\Rightarrow x \in A & \qquad \text{(by def. of converse of } A\text{)}
\end{aligned}
$$

Proof.

We use a double inclusion argument to prove that $A = C$. That is, we will show that $A \subseteq C$ and $C \subseteq A$.

$(C \subseteq A)$: In order to show that $C \subseteq A$, we show that every element of C is an element of A. Namely, $x \in C \Rightarrow x \in A$.

Suppose $x \in C$.

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□

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Thus $C \subseteq A$ by the definition of subset.

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Thus $C \subseteq A$ by the definition of subset.

$(A \subseteq C)$: [Exercise.]

Since $C \subseteq A$ and $A \subseteq C$, we conclude that $A = C$. □