Prove the following.

Theorem

Let A, B, and C be sets where $B = \overline{A}$ and $C = \overline{B}$. Then A = C.

▲ロト ▲帰ト ▲ヨト ▲ヨト 三日 - の々ぐ

That is, we are proving that $A = \overline{\overline{A}}$.

Prove the following.

Theorem

Let A, B, and C be sets where $B = \overline{A}$ and $C = \overline{B}$. Then A = C.

That is, we are proving that $A = \overline{\overline{A}}$. Why is that?

$$C = \overline{B} = \overline{\overline{A}}$$

(replace B with \overline{A} in the last equality)

Prove the following.

Theorem

Let A, B, and C be sets where $B = \overline{A}$ and $C = \overline{B}$. Then A = C.

That is, we are proving that $A = \overline{\overline{A}}$. Why is that?

$$C = \overline{B} = \overline{\overline{A}}$$

(replace *B* with \overline{A} in the last equality) Proving A = C is the same as showing $A = \overline{\overline{A}}$.

Let A, B, and C be sets where $B = \overline{A}$ and $C = \overline{B}$. Then A = C.

Definition (Set equality)

Given
$$A = B$$
, we know
 $\circ \quad x \in A \quad \Rightarrow \quad x \in B \quad \circ$
 $\circ \quad x \in B \quad \Rightarrow \quad x \in A \quad \circ$
 $\circ \quad x \notin A \quad \Rightarrow \quad x \notin B \quad \circ$
 $\circ \quad x \notin B \quad \Rightarrow \quad x \notin A \quad \circ$

Let A, B, and C be sets where $B = \overline{A}$ and $C = \overline{B}$. Then A = C.

Definition (Set equality)



Let A, B, and C be sets where $B = \overline{A}$ and $C = \overline{B}$. Then A = C.

Definition (Set equality)

Given
$$B = \overline{A}$$
, we know
 $\circ \quad x \in B \quad \Rightarrow \quad x \in \overline{A} \quad \circ$
 $\circ \quad x \in \overline{A} \quad \Rightarrow \quad x \in B \quad \circ$
 $\circ \quad x \notin B \quad \Rightarrow \quad x \notin \overline{A} \quad \circ$
 $\circ \quad x \notin \overline{A} \quad \Rightarrow \quad x \notin B \quad \circ$

Let A, B, and C be sets where $B = \overline{A}$ and $C = \overline{B}$. Then A = C.

Definition (Set equality)

Given
$$C = \overline{B}$$
, we know
 $\circ \quad x \in C \quad \Rightarrow \quad x \in \overline{B} \quad \circ$
 $\circ \quad x \in \overline{B} \quad \Rightarrow \quad x \in C \quad \circ$
 $\circ \quad x \notin C \quad \Rightarrow \quad x \notin \overline{B} \quad \circ$
 $\circ \quad x \notin \overline{B} \quad \Rightarrow \quad x \notin C \quad \circ$

Let A, B, and C be sets where $B = \overline{A}$ and $C = \overline{B}$. Then A = C.

Definition (Set complement)

Let A be a set. Then $\overline{A} = \{x : x \notin A\}$.

Ģ	Given a s	et A,	we knov	v
0	$x \in A$	\Rightarrow	$x \notin \overline{A}$	0
0	$x \notin \overline{A}$	\Rightarrow	$x \in A$	0
0	$x \notin A$	\Rightarrow	$x \in \overline{A}$	0
0	$x \in \overline{A}$	\Rightarrow	$x \notin A$	0

< □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > <

Let A, B, and C be sets where $B = \overline{A}$ and $C = \overline{B}$. Then A = C.

Definition (Set complement)

Let A be a set. Then $\overline{A} = \{x : x \notin A\}$.



イロト 不得 トイヨト イヨト

э

Let A, B, and C be sets where $B = \overline{A}$ and $C = \overline{B}$. Then A = C.

Definition (Set complement)

Let A be a set. Then $\overline{A} = \{x : x \notin A\}$.

Given the set A, we know $\circ x \in A \Rightarrow x \notin \overline{A} \circ$ $\circ x \notin \overline{A} \Rightarrow x \in A \circ$ $\circ x \notin A \Rightarrow x \in \overline{A} \circ$ $\circ x \in \overline{A} \Rightarrow x \notin A \circ$

Let A, B, and C be sets where $B = \overline{A}$ and $C = \overline{B}$. Then A = C.

Definition (Set complement)

Let A be a set. Then $\overline{A} = \{x : x \notin A\}$.

Given the set B , we know					
0	$x \in B$	\Rightarrow	$x \notin \overline{B}$	0	
0	$x \notin \overline{B}$	\Rightarrow	$x \in B$	0	
0	$x \notin B$	\Rightarrow	$x \in \overline{B}$	0	
0	$x \in \overline{B}$	\Rightarrow	<i>x</i> ∉ <i>B</i>	0	

< □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > <

Let A, B, and C be sets where $B = \overline{A}$ and $C = \overline{B}$. Then A = C.

Proof.

. . .

We use a double inclusion argument to prove that A = C. That is, we will show that $A \subseteq C$ and $C \subseteq A$.

Theorem (Double inclusion principle)

Let A and B be sets. Then A = B if and only if $A \subseteq B$ and $B \subseteq A$.

Let A, B, and C be sets where $B = \overline{A}$ and $C = \overline{B}$. Then A = C.

Proof.

. . .

We use a double inclusion argument to prove that A = C. That is, we will show that $A \subseteq C$ and $C \subseteq A$.

 $(C \subseteq A)$: In order to show that $C \subseteq A$, we show that every element of C is an element of A. Namely, $x \in C \Rightarrow x \in A$.

Definition (Subset)

Let A and B be sets. Then $A \subseteq B$ if $x \in A \Rightarrow x \in B$.

	Given $B = \overline{A}$,	we know		Given C	$=\overline{B}$,	we know	0	Given the	set A	l, we know	Given the	set E	3, we know
0	$x \in B \Rightarrow$	$x \in \overline{A}$ o	0	$\circ x \in C$	\Rightarrow	$x\in\overline{B}~\circ$	0	$x \in A$	\Rightarrow	$x \notin \overline{A} \circ$	$\circ x \in B$	\Rightarrow	$x\notin \overline{B} \circ$
0	$x \in \overline{A} \Rightarrow$	$x \in B \circ$	0	$\circ x \in \overline{B}$	\Rightarrow	$x \in C$ o	0	$x\not\in\overline{A}$	\Rightarrow	$x \in A$ o	$\circ x \notin \overline{B}$	\Rightarrow	$x\in B$ o
0	$x \notin B \Rightarrow$	$x \notin \overline{A} \circ$	0	• <i>x</i> ∉ C	\Rightarrow	$x \notin \overline{B} ~\circ$	0	$x \notin A$	\Rightarrow	$x \in \overline{A}$ °	o x ∉ B	\Rightarrow	$x\in\overline{B}~\circ$
<	$x \notin \overline{A} \Rightarrow$	x∉B ∘	<	$\circ x \notin \overline{B}$	\Rightarrow	<i>x</i> ∉C ∘	0	$x\in\overline{A}$	\Rightarrow	$x \notin A \circ$	$\circ x \in \overline{B}$	\Rightarrow	x∉B ∘

 $x \in \mathcal{C} \Rightarrow \\ \vdots$

 $\Rightarrow x \in A$

Given $B = \overline{A}$, we know	Given $C = \overline{B}$, we know	Given the set A , we know	Given the set B , we know
$\circ \ x \in B \ \Rightarrow \ x \in \overline{A} \ \circ$	$\circ x \in C \Rightarrow x \in \overline{B} \circ$	$\circ \ x \in A \ \Rightarrow \ x \notin \overline{A} \ \circ$	$\circ \ x \in B \ \Rightarrow \ x \notin \overline{B} \ \circ$
$\circ \ x \in \overline{A} \ \Rightarrow \ x \in B \ \circ$	$\circ \ x \in \overline{B} \ \Rightarrow \ x \in C \ \circ$	$\circ x \notin \overline{A} \Rightarrow x \in A \circ$	$\circ x \notin \overline{B} \Rightarrow x \in B \circ$
$\circ \ x \notin B \ \Rightarrow \ x \notin \overline{A} \ \circ$	$\circ \ x \notin C \ \Rightarrow \ x \notin \overline{B} \ \circ$	$\circ x \notin A \Rightarrow x \in \overline{A} \circ$	$\circ x \notin B \Rightarrow x \in \overline{B} \circ$
$\circ x \notin \overline{A} \Rightarrow x \notin B \circ$	$\circ \ x \notin \overline{B} \ \Rightarrow \ x \notin C \ \circ$	$\circ \ x \in \overline{A} \ \Rightarrow \ x \notin A \ \circ$	$\circ \ x \in \overline{B} \ \Rightarrow \ x \notin B \ \circ$

(since $C = \overline{B}$)

◆□▶ ◆□▶ ◆臣▶ ◆臣▶ 臣 の�?

$$x \in C \Rightarrow x \in \overline{B}$$

$$\vdots$$

$$\Rightarrow x \in A$$

	Given $B = \overline{A}$, we k	know	Given C =	$=\overline{B}$, we know	Given the	set A	l, we know	Given the	set B	, we know
6	$\Rightarrow x \in B \Rightarrow x \in B$	Ā o	$\circ x \in C$	$\Rightarrow x \in \overline{B} \circ$	$\circ x \in A$	\Rightarrow	$x\notin\overline{A}~\circ$	$\circ x \in B$	\Rightarrow	$x \notin \overline{B} \circ$
6	$x \in \overline{A} \Rightarrow x \in$	В 0	$\circ x \in \overline{B}$	$\Rightarrow x \in C \circ$	° ×∉Ā	\Rightarrow	$x \in A$ o	° x∉ B	\Rightarrow	$x \in B$ o
6	$\Rightarrow x \notin B \Rightarrow x \notin$	∉Ā ∘	∘ ×∉ C	$\Rightarrow x \notin \overline{B} \circ$	∘ x∉A	\Rightarrow	$x \in \overline{A}$ °	o x ∉ B	\Rightarrow	$x \in \overline{B}$ °
6	$\Rightarrow x \notin \overline{A} \Rightarrow x \notin$	±B ∘	° x∉ B	$\Rightarrow x \notin C \circ$	$\circ x \in \overline{A}$	\Rightarrow	$x \notin A \circ$	$\circ x \in \overline{B}$	\Rightarrow	×∉B ∘

$$x \in C \Rightarrow x \in \overline{B}$$
$$\Rightarrow x \notin B$$
$$\vdots$$
$$\Rightarrow x \in A$$

(since $C = \overline{B}$) (by def. of converse of *B*)

Given $B = \overline{A}$, we know	Given $C = \overline{B}$, we know	Given the set A , we know Given the set B , we know
$\circ x \in B \Rightarrow x \in \overline{A} \circ$	$\circ \ x \in C \ \Rightarrow \ x \in \overline{B} \ \circ$	$\circ \ x \in A \ \Rightarrow \ x \notin \overline{A} \ \circ \qquad \circ \ x \in B \ \Rightarrow \ x \notin \overline{B}$
$\circ x \in \overline{A} \Rightarrow x \in B \circ$	$\circ \ x \in \overline{B} \ \Rightarrow \ x \in C \ \circ$	$\circ x \notin \overline{A} \Rightarrow x \in A \circ \qquad \circ x \notin \overline{B} \Rightarrow x \in B$
$\circ x \notin B \Rightarrow x \notin \overline{A} \circ$	$\circ x \notin C \Rightarrow x \notin \overline{B} \circ$	$\circ \ x \notin A \ \Rightarrow \ x \in \overline{A} \ \circ \ \circ \ x \notin B \ \Rightarrow \ x \in \overline{B}$
$\circ \ x \notin \overline{A} \ \Rightarrow \ x \notin B \ \circ$	$\circ \ x \notin \overline{B} \ \Rightarrow \ x \notin C \ \circ$	$\circ \ x \in \overline{A} \ \Rightarrow \ x \notin A \ \circ \ \ \circ \ x \in \overline{B} \ \Rightarrow \ x \notin B$

 $x \in C \Rightarrow x \in \overline{B}$ $\Rightarrow x \notin B$ $\Rightarrow x \notin \overline{A}$ \vdots $\Rightarrow x \in A$

(since $C = \overline{B}$) (by def. of converse of B) (since $B = \overline{A}$)

▲ロト ▲帰ト ▲ヨト ▲ヨト 三日 - の々ぐ

Given $B = \overline{A}$, we know	Given $C = \overline{B}$, we know	Given the set A , we know	Given the set B , we know
$\circ x \in B \Rightarrow x \in \overline{A} \circ$	$\circ x \in C \Rightarrow x \in \overline{B} \circ$	$\circ x \in A \Rightarrow x \notin \overline{A} \circ$	$\circ \ x \in B \ \Rightarrow \ x \notin \overline{B} \ \circ$
$\circ \ x \in \overline{A} \ \Rightarrow \ x \in B \ \circ$	$\circ \ x \in \overline{B} \ \Rightarrow \ x \in C \ \circ$	$\circ x \notin \overline{A} \Rightarrow x \in A \circ$	$\circ \ x \notin \overline{B} \ \Rightarrow \ x \in B \ \circ$
$\circ x \notin B \Rightarrow x \notin \overline{A} \circ$	$\circ x \notin C \Rightarrow x \notin \overline{B} \circ$	$\circ x \notin A \Rightarrow x \in \overline{A} \circ$	$\circ x \notin B \Rightarrow x \in \overline{B} \circ$
$\circ \ x \notin \overline{A} \ \Rightarrow \ x \notin B \ \circ$	$\circ \ x \notin \overline{B} \ \Rightarrow \ x \notin C \ \circ$	$\circ \ x \in \overline{A} \ \Rightarrow \ x \notin A \ \circ$	$\circ x \in \overline{B} \Rightarrow x \notin B \circ$

 $x \in C \Rightarrow x \in \overline{B}$ $\Rightarrow x \notin B$ $\Rightarrow x \notin \overline{A}$ $\Rightarrow x \in A$

(since $C = \overline{B}$) (by def. of converse of *B*) (since $B = \overline{A}$) (by def. of converse of *A*)

We use a double inclusion argument to prove that A = C. That is, we will show that $A \subseteq C$ and $C \subseteq A$.

 $(C \subseteq A)$: In order to show that $C \subseteq A$, we show that every element of *C* is an element of *A*. Namely, $x \in C \Rightarrow x \in A$.

Suppose $x \in C$.

We use a double inclusion argument to prove that A = C. That is, we will show that $A \subseteq C$ and $C \subseteq A$.

 $(C \subseteq A)$: In order to show that $C \subseteq A$, we show that every element of *C* is an element of *A*. Namely, $x \in C \Rightarrow x \in A$.

Suppose $x \in C$. Then

 $x \in C \Rightarrow x \in \overline{B}$ (since $C = \overline{B}$) $\Rightarrow x \notin B$ (by def. of converse of B) $\Rightarrow x \notin \overline{A}$ (since $B = \overline{A}$) $\Rightarrow x \in A$ (by def. of converse of A).

We use a double inclusion argument to prove that A = C. That is, we will show that $A \subseteq C$ and $C \subseteq A$.

 $(C \subseteq A)$: In order to show that $C \subseteq A$, we show that every element of *C* is an element of *A*. Namely, $x \in C \Rightarrow x \in A$.

Suppose $x \in C$. Then

$x \in C \Rightarrow x \in \overline{B}$	(since $C = \overline{B}$)
$\Rightarrow x \notin B$	(by def. of converse of B)
$\Rightarrow x \notin \overline{A}$	(since $B = \overline{A}$)
$\Rightarrow x \in A$	(by def. of converse of A).

Thus $C \subseteq A$ by the definition of subset.

We use a double inclusion argument to prove that A = C. That is, we will show that $A \subseteq C$ and $C \subseteq A$.

 $(C \subseteq A)$: In order to show that $C \subseteq A$, we show that every element of *C* is an element of *A*. Namely, $x \in C \Rightarrow x \in A$.

Suppose $x \in C$. Then

$$x \in C \Rightarrow x \in \overline{B}$$
(since $C = \overline{B}$) $\Rightarrow x \notin B$ (by def. of converse of B) $\Rightarrow x \notin \overline{A}$ (since $B = \overline{A}$) $\Rightarrow x \in A$ (by def. of converse of A).

Thus $C \subseteq A$ by the definition of subset.

 $(A \subseteq C)$: [Exercise.]

Since $C \subseteq A$ and $A \subseteq C$, we conclude that A = C.