MATH 2001
PROOF BY INDUCTION

General outline of a proof by induction.

Proof.
1. Base case: verify that the first statement is true.
2. Induction step: show that if the k-th statement is true, then the $(k+1)$-st statement is true.

$\square$

Example 1. Prove that for $7 \mid 4^{3n} - 1$ for every non-negative integer n (i.e. $n = 0, 1, 2, 3, \ldots$).

Proof.

$\square$

Example 2. Prove that $\displaystyle\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.

Proof.

$\square$

Example 3. Let $a_1, a_2, a_3, \ldots$ be a sequence of integers where $a_1 = 3$, $a_2 = 1$, and $a_n = a_{n-2} + a_{n-1}$ for each integer $n \geq 4$. Prove that $1 \leq \frac{a_n}{a_{n-1}} \leq 2$ for each $n \geq 3$.

Proof.

□

Example 4. Let $A_1, A_2, \ldots, A_n$ be sets. Prove that $\overline{A_1 \cap A_2 \cap \cdots \cap A_n} = \overline{A_1} \cup \overline{A_2} \cup \ldots \cup \overline{A_n}$.

Proof.

□

Homework. Four proofs due 6pm on Friday, October 16.

Chapter 10: 6, 7, 18, as well as:

HW #4. Let $a_1, a_2, a_3, \ldots$ be the sequence from Exercise 3. Prove that the sequence $\frac{a_2}{a_1}, \frac{a_4}{a_3}, \frac{a_6}{a_5}, \ldots$ is increasing. In other words, prove that $\frac{a_{2n+2}}{a_{2n+1}} - \frac{a_{2n}}{a_{2n-1}} > 0$ for each $n \in \mathbb{N}$.

Suggested reading: Chapter 10.1.