MATH 2001
QUIZ 6

1. (1 pt) Write your name in the top *right* corner of the page.
2. (2 pts) You proved: if $C \neq \varnothing$ and $B \times C \subseteq C \times D$, then $B \subseteq D$. Why is it necessary that $C \neq \varnothing$? Fill in the blanks with a concrete example that illustrates why the theorem is false when $C = \varnothing$.

   Any choice of B and D where $B \not\subseteq D$ is a correct answer for this problem.

   If $B = \{1\}$, $D = \{2\}$, and $C = \varnothing$. Then $B \times C = \varnothing$, and $C \times D = \varnothing$, but $B \not\subseteq D$.
3. Sketch a proof for the statement: if $B \subseteq C$, then $A \times B \subseteq A \times C$.
   (a) (2 pts) Write a one or two sentence introduction for the proof of this statement.

   Suppose A, B, and C are sets and that $B \subseteq C$. We prove that $A \times B \subseteq A \times C$ by showing that if $(x, y) \in A \times B$, then $(x, y) \in A \times C$.

   (b) (4 pts) Arrange the statements to give an outline for the body of the proof. Justify each implication in the space after each line (e.g. cite a definition).

   $a \Rightarrow c$ (definition of Cartesian product)
   $\Rightarrow d$ (definition of subset, and $B \subseteq C$ (given))
   $\Rightarrow b$ (definition of Cartesian product)

   a. $(x, y) \in A \times B$
   b. $(x, y) \in A \times C$
   c. $x \in A$ and $y \in B$
   d. $x \in A$ and $y \in C$
4. (0.5 pts per blank) Fill in the blanks to complete a proof of the following statement: if $C \neq \varnothing$ and $A \times C \subseteq B \times C$, then $A \subseteq B$.

Proof. Let A, B, and C be sets, where $A \times C \subseteq B \times C$ and $C \neq \varnothing$. We prove that $A \subseteq B$ by showing that if $x \in A$, then $x \in B$.

Suppose $x \in A$. Since $C \neq \varnothing$, the set C contains at least one element; call that element y. Therefore, since $x \in A$ and $y \in C$, we know that $(x, y) \in A \times C$ by the definition of Cartesian product. So, $(x, y) \in B \times C$ since $A \times C \subseteq B \times C$. Hence, by the definition of Cartesian product, we see that $x \in B$ and $y \in C$. Thus we have shown that if $x \in A$, then $x \in B$, and therefore $A \subseteq B$ by the definition of subset. $\square$

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