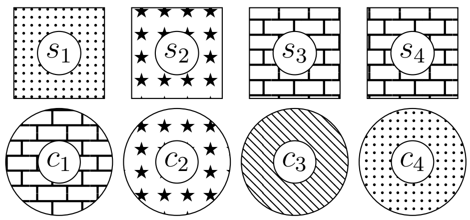
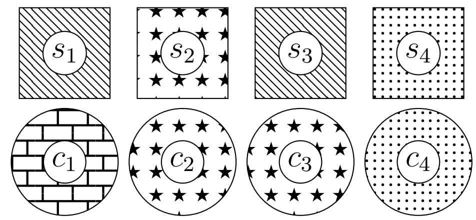
MATH 2001
QUIZ 9

(1) **I.**



II.



Let $P(s_i)$ and $P(c_i)$ denote the pattern on the i-th square and circle, respectively.

- For each of the following statements, write its contrapositive, negation, and converse where requested.
- Additional, identify which diagrams satisfy each statement.

1. [Satisfied by: None] For each i, $P(c_i)$ is "bricks," but $P(s_i)$ is not.

   [I, II] Negation: For all i, $P(c_i)$ is not "bricks" or $P(s_i)$ is "bricks".

2. [II] If there exists an i such that $P(c_i) = P(s_i)$, then there exists a j such that $j \neq i$ and $P(c_j) = P(s_j)$.

   [II] Contrapositive: If there exists an i such that for all j either $j = i$ or $P(c_j) \neq P(s_j)$, then $P(s_i) \neq P(c_i)$.

   [I] Negation: There exists an i such that $P(c_i) = P(s_i)$, and for all j, either $j = i$ or $P(c_j) \neq P(s_j)$.

   [II] Converse: If there exists a j such that $P(c_j) = P(s_j)$, then there exists an i such that $i \neq j$ and $P(s_i) = P(c_i)$.

Date: May 2, 2016.

(2) Edit the following proof for its writing only. We will tackle the logic on the next page.

- Fix all spelling and punctuation errors.
- Mark any grammatical errors. Replace or remove individual words so that the sentences are grammatically correct. Make as few changes as possible and avoid rewriting entire sentences (or the whole proof for that matter.)
- Correct all notational errors relating to mathematical symbols or definitions.

Question: Is $\sqrt{4}$ irrational? Prove your claim.

Proof. We prove (by contradiction) that $\sqrt{4}$ is irrational.

Assume $\sqrt{4}$ is rational , that is ,

$$\sqrt{4} = \frac{a}{b}, \quad \text{where } a, b \in \mathbb{Z}. \tag{1}$$

Assume ~~if~~ [further that] a does not share a common factor with b . ~~because~~ $\cancel{a \mid b}$ [In other words, a/b] is a reduced fraction. Then by clearing denominators and squaring , equation (1) ~~equals~~ [is equivalent to]

$$a^2 = 4b^2. \tag{2}$$

so a^2 is even , and [therefore a is even. Hence] ~~it~~ $a = 2c$, for some $c \in \mathbb{Z}$. ~~Rewording~~ [Substituting] ~~this~~ [$a = 2c$] into ~~the~~ equation [(2) yields]

$$4c^2 = 4b^2, \tag{3}$$

so $c = \pm b$. ~~To get a contradiction see~~ [Note] that a and b share a common factor: c . Specifically , $c \mid b$ because $1 \cdot c = b$, and $c \mid a$ becuase $2 \cdot c = a$. [The fact that] $c \mid a$ and $c \mid b$ ~~which is a contradiction to the statement~~ $\cancel{a \mid b}$ [contradicts the assumption that a/b] is reduced.

Thus ~~proofing~~ ~~that~~ $\sqrt{4}$ is irrational. ∎

The majority of the errors are somewhat minor: word choice, sentence structure, spelling errors. But there are also a number of (what I would consider) significant mistakes.

- Failing to declare that a, b, and c are integers.
- Missing steps and lack of justification, particularly in the middle of the argument.
- Use of ambiguous terms: “*it* equals $2c$. Rewording *this* into *the equation*...”
- Mistaking $a \mid b$ for $\frac{a}{b}$.

All told, there are about eight significant errors and another 20 or so minor ones.

(3) *Proof style:* (circle one) direct contrapositive $\boxed{\text{contradiction}}$

Assumptions: On what assumptions is the author basing his/her argument? (What are the starting assumptions?)

The author assumes that $\sqrt{4}$ is rational, and that $\sqrt{4} = a/b$, where a/b is a fully reduced fraction.

Outline: For each line, cite the appropriate theorem, definition, etc. which justifies the step. If the step is the result of a simple algebraic manipulation, write "alg" for the justification. If the statement does not logically follow from the previous line, write "$*$" for the justification.

$\sqrt{4} = \frac{a}{b}$	$\Rightarrow$	$4b^2 = a^2$	(alg.)
	$\Rightarrow$	a^2 is even	(def. of even, $a^2 = 2(2b^2)$, assuming $a, b \in \mathbb{Z}$)
	$\Rightarrow$	a is even	(proved in class)
	$\Rightarrow$	$a = 2c$	((where $c \in \mathbb{Z}$) def. of even)
	$\Rightarrow$	$4b^2 = 4c^2$	(alg.)
	$\Rightarrow$	$b = c$	(alg. (actually, $b = \pm c$))
$a = 2c$	$\Rightarrow$	$c \mid a$	(def. of divides)
$b = c$	$\Rightarrow$	$c \mid b$	(def. of divides)
$c \mid a$ and $c \mid b$	$\Rightarrow\Leftarrow$		(a and b share a factor, so a/b is not reduced *)

Unjustified statements: For each "$*$", explain why this is a logical gap or gaffe. You do not have to correct the error, just explain why it is an error.

Although the argument is a bit sloppy and light on details, the argument is logically sound. That is, it is sound until the final line. The "contradiction" is not actually a contradiction...