MATH 2001
INTRODUCTION TO SUBSETS

Exercise 1. What is a *subset*? State the exact definition.

Exercise 2. Let A be a set. In your own words, what is the difference between an element of A and a subset of A?

Exercise 3. Let $A = \{1, 2, \{3\}, \{2, 1\}\}$. True (**T**) or false (**F**)?

T	**F**	:	$\varnothing \in A$	**T**	**F**	:	$\varnothing \subseteq A$
T	**F**	:	$3 \in A$	**T**	**F**	:	$3 \subseteq A$
T	**F**	:	$\{1, 2\} \in A$	**T**	**F**	:	$\{1, 2\} \subseteq A$

Are your answers here justified by the statement you gave in Exercise 2? Edit your statement if necessary.

Definition. Let A be a set. The *power set* of A is the set of all subsets of A.

Notation. The power set of A is denoted by $\mathscr{P}(A)$ (`\mathscr P(A)`). In set builder notation,

$$\mathscr{P}(A) = \{X : X \subseteq A\}.$$

"The power set of A is the set of all X, where X is a subset of A."

Exercise 4.

i.) Let $A_0 = \{\}$ be the empty set. Write the set $\mathscr{P}(A_0)$ explicitly. (Remember, $\mathscr{P}(A_0)$ is a set, so use the appropriate set notation.)

ii.) Let $A_1 = \{a\}$. What is $\mathscr{P}(A_1)$?

iii.) Let $A_2 = \{a, b\}$. What is $\mathscr{P}(A_2)$?

iv.) Let A_n be a set of cardinality n. Make a guess as to the cardinality of $\mathscr{P}(A_n)$. In a few sentences, explain how you came about your answer. If you can prove your claim, even better.

Exercise 5. Let A be a set. True (**T**) or false (**F**)?

T	**F**	:	$\varnothing \in \mathscr{P}(A)$	**T**	**F**	:	$\varnothing \subseteq \mathscr{P}(A)$
T	**F**	:	$A \in \mathscr{P}(A)$	**T**	**F**	:	$A \subseteq \mathscr{P}(A)$
T	**F**	:	$\mathscr{P}(A) \in \mathscr{P}(A)$	**T**	**F**	:	$\mathscr{P}(A) \subseteq \mathscr{P}(A)$

Homework. Due Wednesday, January 25 at 2pm.

- Read Sections 1.3 and 1.4 from the text.
- Complete the following exercises (add these to your Overleaf file with the other book problems).
  - Section 1.3: 2, 3, 11, 12.
  - Section 1.4: 5, 14, 17.
  - From this worksheet: Formalize your thoughts from Exercise 4.iv, and write a one paragraph explanation of your guess for the cardinality of $\mathscr{P}(A_n)$.