MATH 2001
INVERSES

Wednesday, April 20: first draft of Proof 15.

Monday, April 25. Book exercises Section 11.4: 6, 7. Final draft of Proof 14.

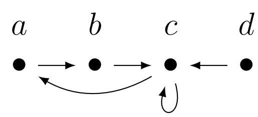
Wednesday, April 27. Book exercises: Section 12.1: 2, 5, 8, 12. Section 12.2: 8, 14. Section 12.5: 3, 6, 8.

Definition. The set R is a relation from the set A to the set B if ...

Definition. Let R be a relation from A to B. The *inverse relation* (denoted R^{-1}) is defined by

$$R^{-1} = \{(b, a) : (a, b) \in R\}.$$

Exercise 1. Let $A = \{a, b, c, d\}$ and consider the relation R from A to itself given by the following diagram. Write the sets R and R^{-1} (using the proper notation).



Definition. Suppose A and B are sets, and f is a relation from A to B. The relation f is a *function* if ...

Exercise 2. Let $A = \{n \in \mathbb{Z} : |n| \leq 2\}$, and let $f : A \to \mathbb{R}$ be defined by $f(x) = x^2$. Write the sets f and f^{-1}. Is f^{-1} a function? Explain why or why not.

Definition. Let $f : A \to B$ be a relation and suppose $U \subseteq A$. Then the *image of* U in B is the set

$$f(U) = \{f(x) \in B : x \in U\}.$$

Definition. Let $f : A \to B$ be a function and suppose $V \subseteq B$. Then the *inverse image of* V in A is the set

$$f^{-1}(V) = \{x \in A : f(x) \in V\}.$$

Exercise 3. Let f and A be defined as in Exercise 2. Write out each of the following sets:

a.) $f(A)$

b.) $f(\mathbb{N} \cap A)$

c.) $f^{-1}(\mathbb{R})$

d.) $f^{-1}(A)$

e.) $f^{-1}([1, 4])$

Exercise 4. Read the following theorem and the accompanying proofs.

Theorem 1. *Suppose $f\colon A \to B$ is a function, and let U and V be subsets of A. Then ...*

Proof. Suppose $x \in U \cup V$. Then $x \in U$ or $x \in V$.[(a)] If $x \in U$, then $f(x) \in f(U)$,[(b)] and so $f(x) \in f(U) \cup f(V)$.[(c)] Otherwise if $x \in V$, then $f(x) \in f(V)$,[(d)] so $f(x) \in f(U) \cup f(V)$.[(e)] Thus we have shown that if $x \in U \cup V$, then $f(x) \in f(U) \cup f(V)$. □

Proof. Suppose $f(x) \in f(U) \cup f(V)$. Then $f(x) \in f(U)$ or $f(x) \in f(V)$.[(f)] If $f(x) \in f(U)$, then $x \in U$, and if $f(x) \in f(V)$, then $x \in V$.[(g)] In either case, $x \in U \cup V$, completing the proof. □

1.) The authors of these proofs have given two rather different arguments.
   (a) What claim has the first author attempted to prove?

   (b) What claim has the second author attempted to prove?

2.) We will now check the validity of the arguments. For each marked line in the proofs above, cite the definition that justifies the statement... unless the statement is unjustifiable, then mark it with $*$.

(a)	(c)	(e)	(g)
(b)	(d)	(f)	

3.) For each starred statement in the previous question (you should have at least one starred statement), explain why the statement is incorrect.

   An error in a proof likely invalidates the proof. Are the claims in the proofs with errors still correct? If so, give a correct proof.