

MATH 2001
PROOF BY INDUCTION

General outline of a proof by induction.

Proof.

1. Base case: verify that the first statement is true.
2. Induction step: show that if the k -th statement is true, then the $(k + 1)$ -st statement is true.

□

Example 1. Prove that $7 \mid 4^{3n} - 1$ for every non-negative integer n (i.e. $n = 0, 1, 2, 3, \dots$).

Proof.

□

Example 2. Prove that $\sum_{i=1}^n i = \frac{n(n+1)}{2}$.

Proof.

□

Example 3. Let a_1, a_2, a_3, \dots be a sequence of integers where $a_1 = 3$, $a_2 = 1$, and $a_n = a_{n-2} + a_{n-1}$ for each integer $n \geq 4$. Prove that $1 \leq \frac{a_n}{a_{n-1}} \leq 2$ for each $n \geq 4$.

Proof.

□

Example 4. Let A_1, A_2, \dots, A_n be sets. Prove that $\overline{A_1 \cap A_2 \cap \dots \cap A_n} = \overline{A_1} \cup \overline{A_2} \cup \dots \cup \overline{A_n}$.

Proof.

□

Suggested proofs:

Chapter 10 (induction): 6, 7, 18.

Chapter 12.6 (functions and inverses): 10, 11, 13.