Hello world!
A First Proof

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Know: Given that $A=B$, we know that

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Show: How would you show that two sets are equal?
If we can show the following:

- if $x \in A$, then $x \in B$, and
- if $x \in B$, then $x \in A$, then we would know that $A=B$.


## Definition

Let $A$ and $B$ be sets, then $A=B$ if $x \in A \Rightarrow x \in B$ and $x \in B \Rightarrow x \in A$.


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## Definition (Subset)

Let $A$ and $B$ be sets, then $A \subseteq B$ if $x \in A \Rightarrow x \in B$.


## Prove the following.

Theorem
Let $A$ and $B$ be sets. If $A=B$, then $A \subseteq B$.
"Knowing that $A$ and $B$ are sets and $A=B$, show that $A \subseteq B$."

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## Proof.

Suppose that $A$ and $B$ are sets, and $A=B$.

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Suppose that $A$ and $B$ are sets, and $A=B$. We prove that $A \subseteq B$ by showing that if $x \in A$, then $x \in B$.
Suppose $x \in A$. By the definition of set equality, if $x \in A$, then $x \in B$.

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"Knowing that $A$ and $B$ are sets and $A=B$, show that $A \subseteq B$."


## Proof.

Suppose that $A$ and $B$ are sets, and $A=B$. We prove that $A \subseteq B$ by showing that if $x \in A$, then $x \in B$.
Suppose $x \in A$. By the definition of set equality, if $x \in A$, then $x \in B$. Thus by the definition of subset, we conclude that $A \subseteq B$.

