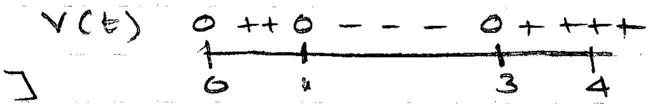
Day 11 Math 131

#1 $v(t) = t^3 - 4t^2 + 3t$ m/s on $[0,4]$

a) Moving forward & backward

$$v(t) = t^3 - 4t^2 + 3t = t(t^2-4t+3+) = t(t-1)(t-3) = 0$$

at $t = 0, 1, 3$



forward on $[0,1]$ and $[3,4]$
backward on $[1,3]$

b) displacement $= \int_0^4 t^3 - 4t^2 + 3t\,dt = \frac{t^4}{4} - \frac{4t^3}{3} + \frac{3t^2}{2}\Big|_0^4$

$$= 64 - \frac{256}{3} + 24 - 0 = \boxed{\frac{8}{3}}\text{ m}$$

c) $v_{ave} = \frac{1}{4-0}\int_0^4 t^3 - 4t^2 + 3t\,dt = \frac{1}{4}(8/3) = \frac{2}{3}$ m/s

d) Dist travelled $= \int_0^3 |t^3 - 4t^2 + 3t|\,dt =$

moving backwards

$$= \int_0^1 t^3 - 4t^2 + 3t\,dt + \int_1^3 -(t^3 - 4t^2 + 3t)\,dt$$

$$= \frac{t^4}{4} - \frac{4t^3}{3} + \frac{3t^2}{2}\Big|_0^1 \overset{\downarrow}{-} \left(\frac{t^4}{4} - \frac{4t^3}{3} + \frac{3t^2}{2}\right)\Big|_1^3$$

$$= \left(\frac{1}{4} - \frac{4}{3} + \frac{3}{2} - 0\right) \overset{\downarrow}{-} \left[\left(\frac{81}{4} - \frac{108}{3} + \frac{27}{2}\right) - \left(\frac{1}{4} - \frac{4}{3} + \frac{3}{2}\right)\right]$$

$$= -\frac{79}{4} + \frac{100}{3} - \frac{21}{2} = \frac{-237 + 400 - 126}{12} = \frac{37}{12} \approx 3.08\bar{3}\text{ m}$$

page 379 #22: $a(t) = e^{-t}$, $v(0) = 60$, $s(0) = 40$

$$v(t) = \int a(t)\,dt = \int e^{-t}\,dt = -e^{-t} + C$$

$$v(0) = 60 = -e^0 + C \Rightarrow C = 61 \Rightarrow \boxed{v(t) = -e^{-t} + 61 \text{ m/s}}$$

$$s(t) = \int v(t)\,dt = \int -e^{-t} + 61\,dt = e^{-t} + 61t + C$$

$$s(0) = e^0 + 61(0) + C = 40 \Rightarrow C = 39. \boxed{s(t) = e^{-t} + 61t + 39\text{ m}}$$

p379 #24 $a(t) = \frac{20}{(t+2)^2}$, $v(0) = 20$, $s(0) = 10$

$u = t+2$
$du = dt$

$$v(t) = \int \frac{20}{(t+2)^2} dt = \int 20(t+2)^{-2} dt = \int 20 u^{-2} du$$

$$= -20 u^{-1} + C = -20(t+2)^{-1} + c$$

$$v(0) = \frac{-20}{2} + C = 20 \Rightarrow C = 30. \quad \text{So } \boxed{v(t) = \frac{-20}{t+2} + 30}$$

$$s(t) = \int \frac{-20}{t+2} + 30\, dt = -20 \ln|t+2| + 30t + c$$

$$s(0) = -20 \ln 2 + 0 + c = 10 \Rightarrow c = 10 + 20 \ln 2$$

$$s(t) = -20 \ln|t+2| + 30t + 10 + 20 \ln 2$$

Honda: $a(t) = k$, constant $v(0) = 0$ ft/s, $v(13) = 88$ ft/s

a) $v(t) = \int k\, dt = kt + c$. $v(0) = 0 = k \cdot 0 + c \Rightarrow c = 0$

Now $v(t) = kt$

$v(13) = k \cdot 13 = 88 \Rightarrow k = \frac{88}{13}$

So $v(t) = \frac{88}{13} t$

b) Notice that $v(t)$ is always non-negative on $0 \le t \le 13$

So

$$\text{Dist travelled} = \int_0^{13} \left|\frac{88}{13} t\right| dt = \int_0^{13} \frac{88}{13} t\, dt$$

$$= \frac{44}{13} t^2 \Big|_0^{13} = 572 \text{ ft.}$$

#4 p380#32: $P'(t) = 5 + 10 \sin\left(\frac{\pi t}{5}\right)$; $P(0) = 35$

$$P(t) = \int 5 + 10 \sin\left(\frac{\pi t}{5}\right) dt = 5t - \frac{50}{\pi} \cos\left(\frac{\pi t}{5}\right) + c$$

$$P(0) = 5(0) - \frac{50}{\pi} \cos(0) + C = 35 \Rightarrow C = 35 + \frac{50}{\pi}$$

$$P(t) = 5t - \frac{50}{\pi} \cos\left(\frac{\pi t}{5}\right) + 35 + \frac{50}{\pi}$$

$$P(15) = 75 - \frac{50}{\pi}(-1) + 35 + \frac{50}{\pi} = 110 + \frac{100}{\pi} \approx 142 \quad (141)$$

$$P(35) = 175 - \frac{50}{\pi}(-1) + 35 + \frac{50}{\pi} = 210 + \frac{100}{\pi} \approx 242 \quad (241)$$

#5

p381 #50(a) $$Q(1) = \overset{\text{empty}}{Q(0)} + \int_0^{\overset{\text{minutes}}{60}} 3\sqrt{t}\, dt = 0 + 2t^{3/2} \Big|_0^{60} = 2(60)^{3/2} \approx 929.51 \text{ liters}$$