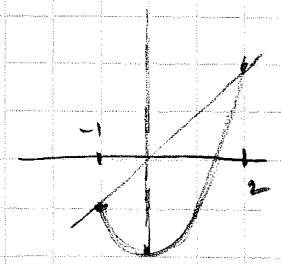
Math 131 Day 12

P 388

#12 Region enclosed by $y = x$ and $y = x^2 - 2$



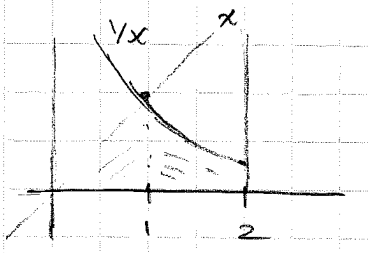
Intersect: $x = x^2 - 2 \Rightarrow x^2 - x - 2 = (x+1)(x-2) = 0$

$x = -1, 2$

$$\text{Area} = \int_{-1}^{2} x - (x^2 - 2)\,dx = \frac{x^2}{2} - \frac{x^3}{3} + 2x \Big|_{-1}^{2}$$

$$= 2 - 8/3 + 4 - (1/2 + 1/3 - 2) = 4\,1/2$$

#17 Enclosed by $y = x$, $y = 1/x$, $y = 0$, $x = 2$

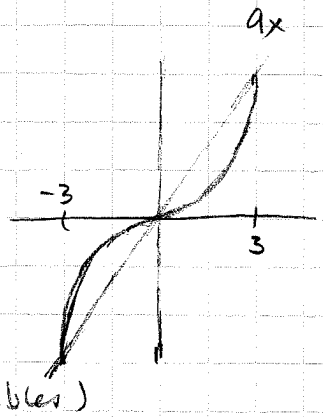


Intersect $x = 1/x \Rightarrow x^2 = 1 \Rightarrow x = 1, -1$

$$\text{Area} = \int_0^1 x\,dx + \int_1^2 1/x\,dx$$

$$= \frac{x^2}{2}\Big|_0^1 + \ln|x|\Big|_1^2 = (1/2 - 0) + (\ln 2 - \ln 1) = 1/2 + \ln 2$$

#20 Enclosed by $y = x^3$ and $y = 9x$



Intersect: $x^3 = 9x \Rightarrow x^3 - 9x = x(x^2 - 9) =$

$= x(x-3)(x+3) = 0$

$x = -3, 0, 3$

$$\text{Area} = \int_{-3}^{0} x^3 - 9x\,dx + \int_0^3 9x - x^3\,dx$$

↘ symmetry (odd... but doubles)

$$= 2\int_0^3 9x - x^3\,dx$$

$$= 2\left[\frac{9}{2}x^2 - \frac{x^4}{4}\Big|_0^3\right] = 2\left(\left(\frac{81}{2} - \frac{81}{4}\right) - 0\right) = \frac{81}{2}$$

#22 Enclosed by $y = x^2(3-x)$ and $y = 12 - 4x$

Intersect:

$x^2(3-x) = 12 - 4x = 4(3-x) \Rightarrow x^2(3-x) = 4(3-x)$

So either $\boxed{x = 3}$ or $x^2 = 4 \Rightarrow \boxed{x = \pm 2}$

Which curve is on top? $x^2(3-x)$ $\qquad$ $12 - 4x$

on $[-2, 2]$ ... at $x = 0$: $0^2(3-0) = 0$ AND $12 - 4(0) = \boxed{12}$

on $[2, 3]$ at 2.5: $(2.5)^2(.5) = \boxed{3.25}$ $\qquad$ $12 - 10 = 2$

$$\text{Area} = \int_{-2}^{2} 12 - 4x - x^2(3-x)\,dx + \int_2^3 x^2(3-x) - (12-4x)\,dx$$

$$= \int_{-2}^{2} 12 - 4x - 3x^2 + x^3\,dx + \int_2^3 3x^2 - x^3 - 12 + 4x\,dx$$

odd / even ↘

$$= 2\int_0^2 12 - 3x^2\,dx + \left(x^3 - \frac{x^4}{4} - 12x + 2x^2\right)\Big|_2^3 =$$

$$= 2(12x - x^3)\Big|_0^2 + (27 - 8\,1/4 - 36 + 18) - (8 - 4 - 24 + 8) = [32] + [3/4]$$

$32\,3/4 = 131/4$