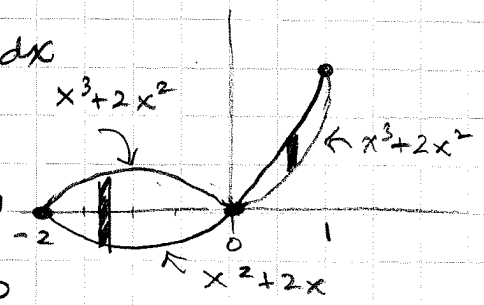
Day 14 Math 131

#1 Area enclosed by $y = x^3 + 2x^2$ and $y = x^2 + 2x$

Intersect: $x^3 + 2x^2 = x^2 + 2x$ or $x^3 + x^2 - 2x = x(x^2 + x - 2)$

$= x(x+2)(x-1) = 0 \quad x = -2, 0, 1$



$$A = \int_{-2}^{0} x^3+2x^2-(x^2+2x)\,dx + \int_{0}^{1} x^2+2x-(x^3+2x^2)\,dx$$

$$= \int_{-2}^{0} x^3+x^2-2x\,dx + \int_{0}^{1} -x^3-x^2+2x\,dx$$

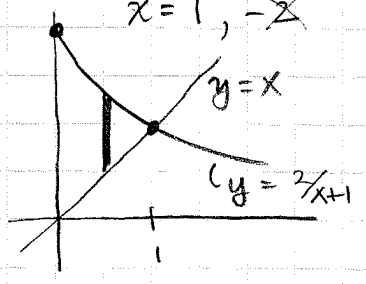
$$= \tfrac{1}{4}x^4 + \tfrac{1}{3}x^3 - x^2 \Big|_{-2}^{0} + \left[-\tfrac{1}{4}x^4 - \tfrac{1}{3}x^3 + x^2\right]\Big|_{0}^{1}$$

$$= 0 - \left[4 - \tfrac{8}{3} - 4\right] + \left[-\tfrac{1}{4} - \tfrac{1}{3} + 1 - (0)\right]$$

$$= 8/3 + 5/12 = \boxed{37/12}$$

#2: Enclosed by $y = x$, $y = \frac{2}{x+1}$, y-axis (1st Quad)

Intersect: $x = 2/x+1 \Rightarrow x^2 + x = 2 \Rightarrow x^2 + x - 2 = (x+2)(x-1) = 0$



$$A = \int_{0}^{1} \frac{2}{x+1} - x\,dx = 2\ln|x+1| - \frac{x^2}{2}\Big|_{0}^{1}$$

$$= 2\ln 2 - 1/2 - 0$$

$$= \underline{2\ln 2 - 1/2}$$

$$= \boxed{\ln 4 - 1/2}$$

aside:

$u = x+1$

$du = dx$

$$\int \frac{2}{x+1}\,dx = \int \frac{2}{u}\,du = 2\ln|u| = 2\ln|x+1| + C$$

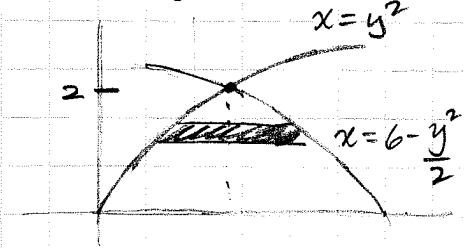
#3: Enclosed by $y = \sqrt{x}$, $y = \sqrt{12-2x}$, x-axis (1st Quad)

Along y-axis

$y = \sqrt{x} \Rightarrow x = y^2$

$y = \sqrt{12-2x} \Rightarrow y^2 = 12 - 2x \Rightarrow x = 6 - \frac{y^2}{2}$

Intersect $y^2 = 6 - \frac{y^2}{2} \Rightarrow \frac{3}{2}y^2 = 6 \Rightarrow y^2 = 4 \Rightarrow y = 2, -2$



$$A = \int_{0}^{2} 6 - \frac{y^2}{2} - y^2\,dy = \int_{0}^{2} 6 - \frac{3}{2}y^2\,dy$$

$$= 6y - \frac{1}{2}y^3\Big|_{0}^{2} = 12 - 4 = \boxed{8}$$

#5) Cross-Section Area = isos $\triangle = \frac{1}{2}bh = \frac{1}{2}x^2 \cdot x^2 = \frac{1}{2}x^4$

$$\text{Vol} = \int_a^b A(x)\,dx = \int_0^2 \frac{1}{2}x^4\,dx = \frac{1}{10}x^5\Big|_0^2 = \frac{32}{10} - 0 = \boxed{\frac{16}{5}}$$

#6) $A(x)$ = Right $\triangle = \frac{1}{2}bh = \frac{1}{2} \cdot x^2 \cdot 2\sqrt{x} = x^{5/2}$

$$\text{Vol} = \int_a^b A(x)\,dx = \int_0^4 x^{5/2}\,dx = \frac{2}{7}x^{7/2}\Big|_0^4 = \boxed{\frac{256}{7}} - 0$$

⇒ #4) Area enclosed by $y = \pi/2$, $y = \arcsin x$, y-axis (1st Quad)

$y = \arcsin x \Rightarrow x = \sin y$

$$A = \int_0^{\pi/2} \sin y\,dy = -\cos y\Big|_0^{\pi/2} = 0 - (-1) = \boxed{1}$$