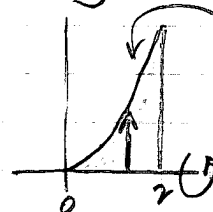
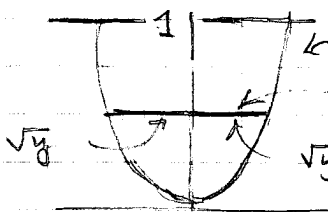
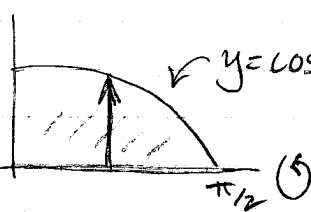
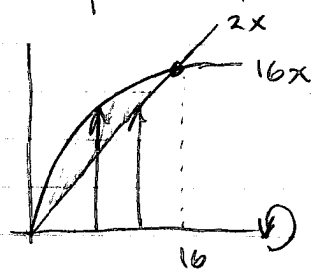


Day 15 Math 131

#1  $y = x^3 = \text{radius}$ $V = \int_a^b \pi (f(x))^2 dx = \pi \int_0^2 (x^3)^2 dx = \pi \int_0^2 x^6 dx$
 $= \frac{\pi}{7} x^7 \Big|_0^2 = \boxed{\frac{128\pi}{7}} \text{ units}^3$

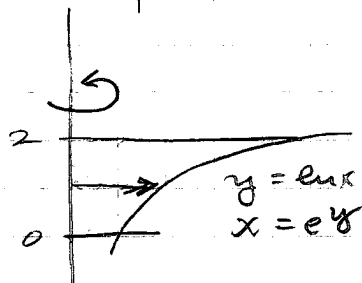
#2 p400 #10  $y = x^2 \Rightarrow x = \sqrt{y}$ $A(y) = \text{square} = (\text{edge})^2 = (2\sqrt{y})^2 = 4y$
 $V = \int_0^1 A(y) dy = \int_0^1 4y dy$
 $= 2y^2 \Big|_0^1 = 2 \text{ units}^3$

#3 p400 #18  $y = \cos x$ $A(x) = \text{circle} = \pi r^2 = \pi (\cos x)^2$
 $V = \int_0^{\pi/2} \pi (\cos x)^2 dx = \pi \int_0^{\pi/2} \frac{1}{2} (1 + \cos 2x) dx$
 $= \frac{\pi}{2} (x + \frac{1}{2} \sin 2x) \Big|_0^{\pi/2}$
 $= \frac{\pi}{2} (\frac{\pi}{2} + 0) - 0 = \boxed{\frac{\pi^2}{4}} \text{ units}^3$

#4 p401 #24  $2x$ $16x^{1/4}$
 Intersect: $16x^{1/4} = 2x \Rightarrow 8x^{1/4} = x$
 So $8^4 x = x^4 \Rightarrow x(x^3 - 8^4) = 0 \Rightarrow x=0, x^3=16$
 $\text{Vol} = \text{Outer} - \text{Inner}$
 $= \int_0^{16} \pi (16x^{1/4})^2 dx - \int_0^{16} \pi (2x)^2 dx$
 $= \int_0^{16} 256\pi x^{1/2} dx - \int_0^{16} 4\pi x^2 dx$
 $= \frac{512\pi}{3} x^{3/2} \Big|_0^{16} - \frac{4\pi}{3} x^3 \Big|_0^{16}$
 $= \frac{32768\pi}{3} - \frac{16384\pi}{3} = \frac{16384\pi}{3} = \boxed{5461 \frac{1}{3} \pi}$

Math 131 Day 15

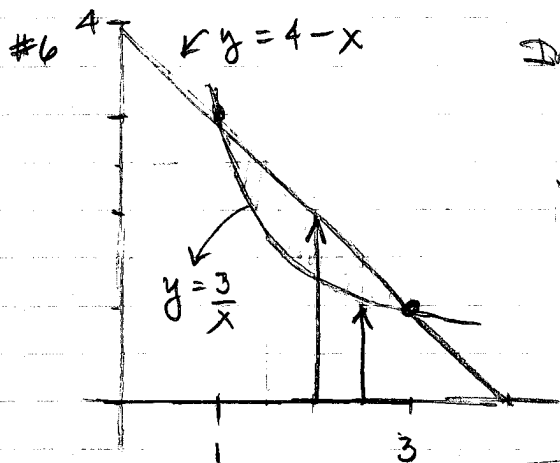
#5 p401 #32



$$\text{Cross-Area} = A(y) = \pi (e^y)^2 = \pi e^{2y}$$

$$V = \int_0^2 A(y) dy = \int_0^2 \pi e^{2y} dy$$

$$= \frac{\pi}{2} e^{2y} \Big|_0^2 = \frac{\pi}{2} [e^4 - 1] \text{ units}^3$$



Intersect: $\frac{3}{x} = 4 - x \Rightarrow 3 = 4x - x^2$
 $x^2 - 4x + 3 = (x-1)(x-3) = 0 \quad x = 1, 3$

$$V = \text{Outer} - \text{Inner}$$

$$= \int_1^3 \pi (4-x)^2 dx - \int_1^3 \pi \left(\frac{3}{x}\right)^2 dx$$

$$= \pi \int_1^3 (16 - 8x + x^2) dx - \pi \int_1^3 9x^{-2} dx$$

$$= \pi (16x - 4x^2 + \frac{x^3}{3}) \Big|_1^3 - \pi (-9x^{-1}) \Big|_1^3$$

$$= \pi (48 - 36 + 9) - (16 - 4 + \frac{1}{3}) - \pi [-3 - (-9)]$$

$$= 8\frac{2}{3}\pi - 6\pi = \frac{8}{3}\pi \text{ units}^3$$

$\downarrow \frac{(4-x)^3}{3} \text{ etc}$
 $\frac{1}{2}$

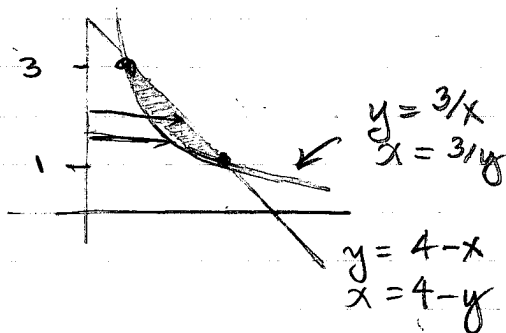
Around y-axis: $y = 3/x \Rightarrow x = 3/y$
 $y = 4 - x \Rightarrow x = 4 - y$

Intersections: when $x=1 \Rightarrow y=3$; when $x=3 \Rightarrow y=1$

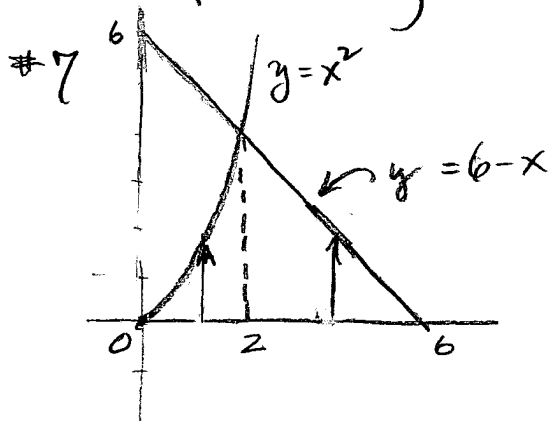
$$V = \text{Outer} - \text{Inner} = \int_1^3 \pi (4-y)^2 dy - \int_1^3 \pi \left(\frac{3}{y}\right)^2 dy$$

SAME AS ABOVE!

$$= \frac{8\pi}{3} \text{ units}^3$$



Math 131 Day 15



Intersect: $x^2 = 6 - x$
 $x^2 + x - 6 = (x-2)(x+3) = 0$
 $x = 2, -3$

$$\begin{aligned}
 Vol &= \int_0^2 \pi (x^2)^2 dx + \int_2^6 \pi (6-x)^2 dx \\
 &= \int_0^2 \pi x^4 dx + \int_4^0 \pi u^2 du \quad \begin{array}{l} u = 6-x \\ du = -dx \\ -du = dx \\ x=2 \Rightarrow u=4 \\ x=6 \Rightarrow u=0 \end{array} \\
 &= \left. \frac{\pi x^5}{5} \right|_0^2 + \int_0^4 \pi u^2 du \\
 &= \frac{\pi \cdot 32}{5} + \left. \frac{\pi \cdot u^3}{3} \right|_0^4 \\
 &= \frac{32\pi}{5} + \frac{64\pi}{3} = \boxed{\frac{416\pi}{15}}
 \end{aligned}$$