Math 131 Day 26

#1a $\int \frac{2}{x^3-x}\,dx = \int \frac{1}{x-1} + \frac{1}{x+1} - \frac{2}{x}\,dx = \ln|x-1| + \ln|x+1| - 2\ln|x| + c$

$$\frac{2}{x^3-x} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x} = \frac{Ax^2 + Ax + Bx^2 - Bx + Cx^2 - C}{x^3-x}$$

x^2 : $A + B + C = 0$

x : $A - B = 0$

const : $-C = 2 \Rightarrow C = -2,\ A = 1,\ B = 1$

b) $\int \frac{2x^2}{(x-1)^2(x+1)}\,dx$ | So $= \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1} = \frac{Ax^2 - A + Bx + B + Cx^2 - 2Cx + C}{(x-1)^2(x+1)}$

x^2 : $A + C = 2$

x : $B - 2C = 0$

Const $\underline{-A + B + C = 0}$

$0 + 2B + 0 = 2 \Rightarrow B = 1,\ C = 1/2,\ A = 3/2$

So

$$\int \frac{2x^2}{(x-1)^2(x+1)}\,dx = \int \frac{3/2}{x-1} + \frac{1}{(x-1)^2} + \frac{1/2}{x+1}\,dx$$

$$= 3/2 \ln|x-1| - (x-1)^{-1} + 1/2 \ln|x+1| + c$$

$\int \frac{-4x+4}{(x-2)^2 x}\,dx$

$$\frac{-4x+4}{(x-2)^2 x} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{x} = \frac{Ax^2 - 2Ax + Bx + Cx^2 - 4Cx + 4C}{(x-2)^2 x}$$

x^2 : $A + C = 0$

x : $-2A + B - 4C = -4$

Const $4C = 4 \Rightarrow C = 1,\ A = -1,\ B = -2$

So $\int \frac{-4x+4}{(x-2)^2 x}\,dx = \int \frac{-1}{x-2} - \frac{2}{(x-2)^2} + \frac{1}{x}\,dx$

$$= -\ln|x-2| + 2(x-2)^{-1} + \ln|x| + C$$

#2

a) $$\lim_{x \to -1} \frac{x^2 - 2x - 3}{x+1} \overset{L'H}{=} \lim_{x \to -1} \frac{2x-2}{1} = \boxed{-4}$$

b) $$\lim_{x \to 1} \frac{\ln(x^2)}{x^2 - 1} \overset{L'H}{=} \lim_{x \to 1} \frac{\frac{2}{x}}{2x} = \frac{2}{2} = \boxed{1}$$

c $$\lim_{x \to 0} \frac{\sin(ax)}{\sin(bx)} \overset{L'H}{=} \lim_{x \to 0} \frac{a\cos(ax)}{b\cos(bx)} = \boxed{\frac{a}{b}}$$

d) $$\lim_{x \to 0} \frac{x}{\arctan(2x)} = \lim_{x \to 0} \frac{1}{\frac{2}{1+4x^2}} = \boxed{\frac{1}{2}}$$

e) $$\lim_{x \to 1} \frac{(\ln x)^2}{x^3} = 0$$ (L'H does not apply)

f) $$\lim_{x \to 0} \frac{\sin^2(3x)}{x^2} = \lim_{x \to 0} \frac{2 \cdot 3 \cdot \sin(3x)\cos(3x)}{2x}$$

$$\overset{L'H}{=} \lim_{x \to 0} \frac{9\left[\cos^2(3x) - \sin^2(3x)\right]}{1}$$

$$= \boxed{9}$$

g) $$\lim_{x \to 1} \frac{x^n - 1}{x - 1} \overset{L'H}{=} \lim_{x \to 1} \frac{n x^{n-1}}{1} = \boxed{n}$$