Math 131 Day 34

My Office Hours: M & W 12:30–2:00, Tu 2:30–4:00, & F 1:15-2:30 or by appointment. **Math Intern** Sun: 12–6pm; M 3-10pm; Tu 2–6, 7–10pm; W and Th: 5-10 pm in Lansing 310. Website: `http://math.hws.edu/~mitchell/Math131S13/index.html`.

Reading and Practice

1. Read and review all of Section 8.5 on the ratio, root, and comparison tests.

Some of these will be Hand In for Monday

Justify your answers with an argument. Make sure you explain why the series you are comparing to either converges or diverges.

1. Page 567 #28.

2. Page 567 #30.

3. $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^{3/2}+1}$.

4. $\displaystyle\sum_{k=1}^{\infty} \frac{1}{3k-\sqrt{k}}$.

5. Page 567 #42.

6. Page 567 #68(a). Consider $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k}$.

Six Tests

1. Direct Comparison Test. Assume $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are series with positive terms.

a) If for all n, we have $0 < a_n \le b_n$ and $\displaystyle\sum_{n=1}^{\infty} b_n$ converges, then $\displaystyle\sum_{n=1}^{\infty} a_n$ converges. (If the bigger series converges, so does the smaller series.)

b) If for all n, we have $0 < b_n \le a_n$ and $\displaystyle\sum_{n=1}^{\infty} b_n$ diverges, then $\displaystyle\sum_{n=1}^{\infty} a_n$ diverges. (If the smaller series diverges, so does the bigger series.)

2. Limit Comparison Test. Assume that $a_n > 0$ and $b_n > 0$ for all n (or at least all $n \ge k$) and that $\displaystyle\lim_{n\to\infty} \frac{a_n}{b_n} = L$.

1) If $0 < L < \infty$ (i.e., L is a positive, *finite* number), then either $\displaystyle\sum_{n=1}^{\infty} a_n$ and $\displaystyle\sum_{n=1}^{\infty} b_n$ *both* converge or *both* diverge.

2) If $L = 0$ and $\displaystyle\sum_{n=1}^{\infty} b_n$ converges, then $\displaystyle\sum_{n=1}^{\infty} a_n$ converges.

3) If $L = \infty$ and $\displaystyle\sum_{n=1}^{\infty} b_n$ diverges, then $\displaystyle\sum_{n=1}^{\infty} a_n$ diverges.

3. The Geometric Series Test.

a) If $|r| < 1$, then the geometric series $\displaystyle\sum_{n=0}^{\infty} ar^n$ converges to $\displaystyle\frac{a}{1-r}$.

b) If $|r| \ge 1$, then the geometric series $\displaystyle\sum_{n=0}^{\infty} ar^n$ diverges.

4. The nth term test for Divergence. If $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ diverges. (If $\lim_{n\to\infty} a_n = 0$, this test is useless.)

5. The Integral Test. If $f(x)$ is a **positive**, **continuous**, and **decreasing** for $x \geq 1$ and $f(n) = a_n$, then $\displaystyle\sum_{n=1}^{\infty} a_n$ and $\displaystyle\int_1^{\infty} f(x)\,dx$ either both converge or both diverge.

6. The p-series Test. The p-series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^p} \begin{cases} \text{converges if } p > 1 \\ \text{diverges if } p \leq 1. \end{cases}$

Class Exercise: Rate These Arguments

Each of the following statements is an attempt to show that a given series is convergent or divergent using the Comparison Test. Classify each statement, 'correct' if the argument is valid, or 'incorrect' if any part of the argument is flawed. (Note: Even if the conclusion is true but the argument that led to it was wrong, classify it as incorrect.)

a) For all $n \geq 3$ we have $\ln n > 1$, so $0 \leq \frac{1}{n} < \frac{1}{n\ln(n)}$, and the series $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n}$ diverges by the p-series test ($p = 1$), so by the Comparison Test, the series $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n\ln(n)}$ diverges.

b) For all $n \geq 1$ we have $\sqrt{n+1} > 1$, so $0 < \frac{1}{n} < \frac{\sqrt{n+1}}{n}$ and the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ diverges by the p-series test ($p = 1$), so by the Comparison Test, the series $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n+1}}{n}$ diverges.

c) For all $n > 2$, $\quad 0 < \frac{n}{3-n^3} < \frac{1}{n^2}$, and the series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^2}$ converges by the p-series test ($p = 2 > 1$), so by the Comparison Test, the series $\displaystyle\sum_{n=2}^{\infty} \frac{n}{3-n^3}$ converges.

d) For all $n \geq 1$, $\quad 0 < \frac{\cos^2(n)}{n^3} < \frac{1}{n^3}$, and the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3}$ converges by the p-series tes ($p = 3 > 1$)t, so by the Comparison Test, the series $\displaystyle\sum_{n=1}^{\infty} \frac{\cos^2(n)}{n^3}$ converges.

e) For all $n \geq 1$, $\quad 0 < \frac{1}{n^2} < \frac{2n+1}{n^3}$, and the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges by the p-series test ($p = 2 > 1$), so by the Comparison Test, the series $\displaystyle\sum_{n=1}^{\infty} \frac{2n+1}{n^3}$ converges.