

*This homework covers reductions to show hardness. It is due in class Monday, May 5.*

*Write your solutions carefully — your work should be neat, readable, organized, and polished.*

*See the Policies page on the course website for information about revise-and-resubmit, late work, and academic integrity as it applies to homework.*

1. The vertex cover problem is known to be NP-complete. Prove that the card collector problem is also NP-complete using a reduction. Keep in mind that you need to show that the card collector problem is in NP as well as giving the reduction itself. Also explain why the reduction is correct — you'll get the right answer to the problem — and polynomial-time.

The vertex cover and card collector problems are described below.

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**Vertex cover** Given a graph  $G$  and a positive integer  $k$ , is there a vertex cover of  $G$  with size at most  $k$ ? A *vertex cover* of a graph  $G$  is a set of vertices such that every edge of  $G$  has at least one endpoint in the set.

**Card collector** Given  $n$  packets of cards (each of which contains a subset of the available cards) and a positive integer  $k$ , is it possible to collect the full set of cards by buying no more than  $k$  packets? For example, if there are four cards available (A, B, C, D) and the packets contain  $\{A,B\}$ ,  $\{B,C\}$ ,  $\{D\}$ ,  $\{B,D\}$ , then the answer is *no* for  $k=2$  (packet 1 is needed for card A, packet 2 is needed for card C, and either packet 3 or 4 is needed for card D) and *yes* for  $k=3$  (e.g. packets  $\{A,B\}$ ,  $\{B,C\}$ ,  $\{D\}$  result in the full set).