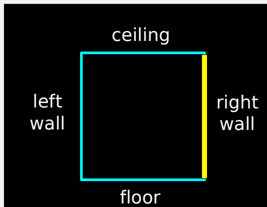
Solving the Rendering Equation

- path tracing [1986] is one algorithm for generating approximate solutions to the rendering equation
  - *unbiased* – correct (no systematic error) but computationally expensive to reduce noise to a sufficiently low level
  - often used to generate reference images for testing other renderers
- there are others
  - e.g. photon mapping [1995]
    - two-pass algorithm – traces rays from camera and rays from light sources
    - Monte Carlo method
  - e.g. radiosity [1984]
    - finite element method (numerical solution) based on simulating energy transfer
    - viewpoint independent
    - only accounts for diffuse reflection

Radiosity

- *radiosity* is the rate at which energy leaves a surface
  - sum of rate at which surface emits energy and at which it reflects incident energy
- the idea: simulate the energy transfer between diffuse surfaces in the scene
  - subdivide scene into little patches and consider how each pair of patches send and receive light energy
    - mathematics based on thermal engineering models of emission and reflection
    - assumes conservation of energy in closed environments
    - an approximation! (patches should be infinitely small)
  - brightness (color) of surface in rendered scene is proportional to its radiosity
- notes
  - only captures diffuse reflection
  - method is view independent

Radiosity Example



due to the geometry of the room,
1/4 of the energy leaving a surface reaches an adjacent surface
1/2 of the energy leaving a surface reaches the opposite surface

form factors

the left wall and ceiling are painted white, and each reflect 75% of the energy hitting them
the floor is carpeted with a dark carpet, and reflects 10% of the energy hitting it
the right wall is a window, and reflects 1% of the energy hitting it

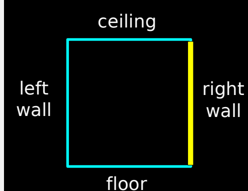
material properties

all light sources are area lights

the right wall emits one unit of light

Radiosity Example: Equations

B_{surf} = radiosity of surface *surf* (watts/m²)
= sum of light emitted and light reflected



light reaching surface = sum of the product of the light leaving other surfaces and the fraction reaching this surface

$$B_{ceiling} = 0 + .75\left(.25\,B_{rwall} + .25\,B_{lwall} + .5\,B_{floor}\right)$$
$$B_{floor} = 0 + .10\left(.25\,B_{rwall} + .25\,B_{lwall} + .5\,B_{ceiling}\right)$$
$$B_{lwall} = 0 + .75\left(.25\,B_{ceiling} + .25\,B_{floor} + .5\,B_{rwall}\right)$$
$$B_{rwall} = 1 + .01\left(.25\,B_{ceiling} + .25\,B_{floor} + .5\,B_{lwall}\right)$$

light emitted by this surface

fraction of light reflected by this surface

General Radiosity Equation

$$B_k = E_k + \rho_k \sum_{j=1}^{n} \frac{B_j A_j F_{jk}}{A_k}$$

for a scene with *n* patches

B_k = total radiosity for patch *k* (watts/m²)

E_k = light emitted from patch *k* (watts/m²)

ρ_k = reflectivity factor (fraction of incident energy reflected by patch *k*, $0 \le \rho_k \le 1$)

$B_j A_j$ = total energy radiated by patch *j* with area A_j

F_{jk} = form factor (fraction of energy leaving patch *j* that arrives at patch *k*)

$(B_j A_j F_{jk})/A_k$ = energy leaving patch *j* and arriving at patch *k* per unit area of patch *k*

n equations, *n* unknowns ... can solve!

Solving the Radiosity Equations

$$B_k = E_k + \rho_k \sum_{j=1}^{n} \frac{B_j A_j F_{jk}}{A_k}$$

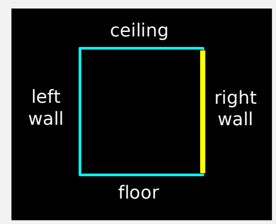
- strategy #1: apply numerical techniques for solving systems of equations
  - large scenes can easily have 10,000+ patches
  - requires a lot of computation to solve 10,000 simultaneous equations and to compute 10^8 form factors
    - (though we haven't said how that is done yet)
  - requires a lot of *memory* to solve 10,000 simultaneous equations and to store 10^8 form factors

Solving the Radiosity Equations

$$B_k = E_k + \rho_k \sum_{j=1}^{n} \frac{B_j A_j F_{jk}}{A_k}$$

- strategy #2: gathering
  - iteratively compute energy reaching each patch
  - allows for progressive refinement

Gathering Example I



due to the geometry of the room,
1/4 of the energy leaving a surface reaches an adjacent surface
1/2 of the energy leaving a surface reaches the opposite surface

the left wall and ceiling are painted white, and each reflect 75% of the energy hitting them
the floor is carpeted with a dark carpet, and reflects 10% of the energy hitting it
the right wall is a window, and reflects 1% of the energy hitting it

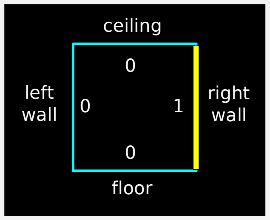
the right wall emits one unit of light

current radiosities

right wall	1
left wall	0
floor	0
ceiling	0

Gathering Example II

in each round, compute how much light is received by each patch based on current radiosities



1/4 of the energy reaches adjacent
1/2 of the energy reaches opposite

left wall and ceiling reflect 75%
floor reflects 10%
right wall reflects 1%

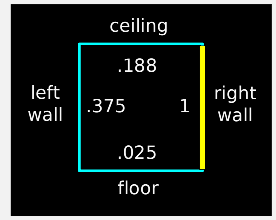
how much light is received by...
- the floor?
  (1/4)(1)+(1/4)(0)+(1/2)(0) = 1/4
- the left wall?
  (1/2)(1)+(1/4)(0)+(1/4)(0) = 1/2
- the ceiling?
  (1/4)(1)+(1/4)(0)+(1/2)(0) = 1/4
- the right wall?
  (1/4)(0)+(1/4)(0)+(1/2)(0) = 0

new radiosities

right wall
  1+(.01)(0) = 1
left wall
  0+(.75)(.5) = .375
floor
  0+(.1)(.25) = .025
ceiling
  0+(.75)(.25) = .188

Gathering via Example III

in each round, compute how much light is received by each patch based on current radiosities



new radiosities

right wall
  1+(.01)(.241) = 1.002
left wall
  0+(.75)(.553) = .415
floor
  0+(.1)(.438) = .044
ceiling
  0+(.75)(.356) = .267

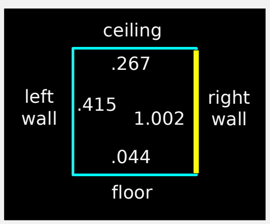
how much light is received by...
- the floor?
  (1/4)(1)+(1/4)(.375)+(1/2)(.188) = .438
- the left wall?
  (1/2)(1)+(1/4)(.025)+(1/4)(.188) = .553
- the ceiling?
  (1/4)(1)+(1/4)(.375)+(1/2)(.025) = .356
- the right wall?
  (1/4)(.188)+(1/4)(.025)+(1/2)(.375) = .241

1/4 of the energy reaches adjacent
1/2 of the energy reaches opposite

left wall, ceiling reflect 75%
floor reflects 10%
right wall reflects 1%

Gathering via Example IV

in each round, compute how much light is received by each patch based on current radiosities



new radiosities

right wall
  1+(.01)(.285) = 1.003
left wall
  0+(.75)(.579) = .434
floor
  0+(.1)(.488) = .049
ceiling
  0+(.75)(.376) = .282

how much light is received by...
- the floor?
  (1/4)(1.002)+(1/4)(.415)+(1/2)(.267) = .488
- the left wall?
  (1/2)(1.002)+(1/4)(.044)+(1/4)(.267) = .579
- the ceiling?
  (1/4)(1.002)+(1/4)(.415)+(1/2)(.044) = .376
- the right wall?
  (1/4)(.267)+(1/4)(.044)+(1/2)(.415) = .285

1/4 of the energy reaches adjacent
1/2 of the energy reaches opposite

left wall, ceiling reflect 75%
floor reflects 10%
right wall reflects 1%

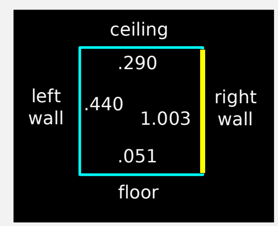
Gathering via Example V

in each round, compute how much light is received by each patch based on current radiosities

and so forth...

	radiosity				energy received			
step	floor	left wall	ceiling	right wall	floor	left wall	ceiling	right wall
1	0.000	0.000	0.000	1.000	0.250	0.500	0.250	0.000
2	0.025	0.375	0.188	1.000	0.438	0.553	0.356	0.241
3	0.044	0.415	0.267	1.002	0.488	0.579	0.376	0.285
4	0.049	0.434	0.282	1.003	0.500	0.584	0.384	0.300
5	0.050	0.438	0.288	1.003	0.504	0.586	0.385	0.304
6	0.050	0.439	0.289	1.003	0.505	0.586	0.386	0.305
7	0.051	0.440	0.289	1.003	0.505	0.586	0.386	0.305
8	0.051	0.440	0.289	1.003	0.505	0.587	0.386	0.305
9	0.051	0.440	0.289	1.003	0.505	0.587	0.386	0.305
10	0.051	0.440	0.290	1.003	0.505	0.587	0.386	0.305

final result

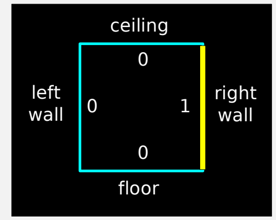


Solving the Radiosity Equations

$$B_k = E_k + \rho_k \sum_{j=1}^{n} \frac{B_j A_j F_{jk}}{A_k}$$

- strategy #3: shooting
  - iteratively send energy from one patch at a time to other patches
    - choose patch with highest unshot radiosity
  - allows for progressive refinement
  - converges faster than gathering
  - requires less data resident in memory at one time

Shooting Example I



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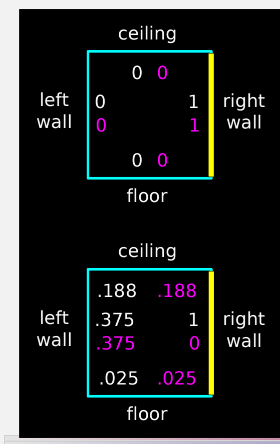
the right wall emits one unit of light

Shooting Example II



choose patch with highest unshot radiosity and distribute to other patches, adding to their radiosities

current radiosity
unshot radiosity

shoot from the right wall...

ceiling
receives (1/4)(1) = .25
reflects back (.75)(.25) = .188
left wall
receives (1/2)(1) = .5
reflects back (.75)(.5) = .375
floor
receives (1/4)(1) = .25
reflects back (.1)(.25) = .025

Shooting Example III

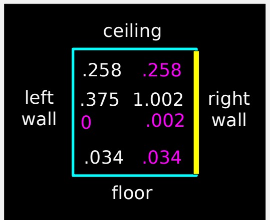
choose patch with highest unshot radiosity and distribute to other patches, adding to their radiosities

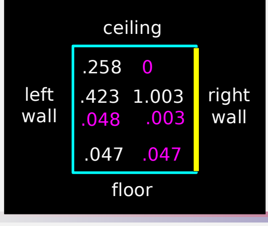
current radiosity
unshot radiosity

shoot from the left wall...

ceiling
receives (1/4)(.375) = .094
reflects back (.75)(.094) = .07
right wall
receives (1/2)(.375) = .188
reflects back (.01)(.188) = .002
floor
receives (1/4)(.375) = .094
reflects back (.1)(.094) = .009

Shooting Example IV





choose patch with highest unshot radiosity and distribute to other patches, adding to their radiosities

current radiosity
unshot radiosity

shoot from the ceiling...

left wall
- receives (1/4)(.258) = .065
- reflects back (.75)(.065) = .048

right wall
- receives (1/4)(.258) = .065
- reflects back (.01)(.065) = .001

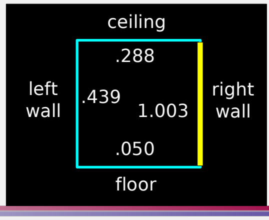
floor
- receives (1/2)(.258) = .129
- reflects back (.1)(.129) = .013

Shooting Example V

and so forth...

	total radiosity				unshot radiosity				new radiosity			
step	floor	left wall	ceiling	right wall	floor	left wall	ceiling	right wall	floor	left wall	ceiling	right wall
1	0.000	0.000	0.000	1.000	0.000	0.000	0.000	1.000	0.025	0.375	0.188	0.000
2	0.025	0.375	0.188	1.000	0.025	0.375	0.188	0.000	0.009	0.000	0.070	0.002
3	0.034	0.375	0.258	1.002	0.034	0.000	0.258	0.002	0.013	0.048	0.000	0.001
4	0.047	0.423	0.258	1.003	0.047	0.048	0.000	0.003	0.001	0.000	0.009	0.000
5	0.048	0.423	0.267	1.003	0.048	0.000	0.009	0.003	0.000	0.009	0.018	0.000
6	0.048	0.432	0.285	1.003	0.000	0.009	0.027	0.003	0.001	0.005	0.000	0.000
7	0.050	0.438	0.285	1.003	0.001	0.014	0.000	0.003	0.000	0.000	0.003	0.000
8	0.050	0.438	0.288	1.003	0.002	0.000	0.003	0.003	0.000	0.000	0.000	0.000
9	0.050	0.438	0.288	1.003	0.002	0.000	0.000	0.003	0.000	0.001	0.001	0.000
10	0.050	0.439	0.288	1.003	0.002	0.002	0.001	0.000	0.000	0.000	0.001	0.000

final result



Solving the Radiosity Equations

$$B_k = E_k + \rho_k \sum_{j=1}^{n} \frac{B_j A_j F_{jk}}{A_k}$$

- strategy #4: importance-driven radiosity
  - shoot from patch which contributes most to the scene
  - allows for progressive refinement
  - converges faster than shooting
  - view dependent

Computing Form Factors

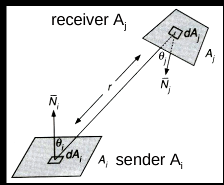
- form factor for very small sender area dA_i and very small receiver area dA_j

$$dF_{di,dj} = \frac{\cos\theta_i \cos\theta_j}{\pi r^2} H_{ij}\, dA_j$$

r is distance between patches

θ_i and θ_j are angles between the ray between the patches and the patch surface normals

H_{ij} = 1 if dA_j is visible from dA_i, 0 otherwise



from "Computer Graphics: Principles and Practice", Foley, van Dam, Feiner, Hughes

- takes into account patch separation, relative orientation, occlusion, conservation of energy

Radiosity Summary

- advantages
  - nice results when there are many diffuse surfaces
  - obtain proper color bleeding effects
  - area light sources produce soft shadows
  - no ambient term hack – for low lighting, just adjust the light intensities
  - view-independent (except in some variations)
- limitations
  - many assumptions!
    - assumes radiation from light source is uniform in all directions
    - assumes intermediate medium is non-participatory
    - assumes opaque surfaces
    - assumes no diffraction
  - matrices are huge – very computationally and memory intensive
  - computation of form factors can be difficult
  - does not capture specular reflection

Radiosity and Raytracing

- radiosity is good for diffuse surfaces but poor for specular reflections
- raytracing is good for specular reflections but poor for diffuse surfaces
- but simply adding radiosity and raytracing intensities doesn't (quite) work…
  - raytrace first, then apply radiosity with highlights treated as new lights
  - apply radiosity first, then raytrace with each point treated as a light source emitting its radiosity
- in both cases, blend colors obtained from each pass