Math 110. Mindscapes for Week 4. Names: ____________________

Instructions: *You must work with a partner on these.* Use these sheets (staple). Hand in only one good copy for you both. Keep a copy for yourself.

1. Topsy turvy. We have been working with the **Fibonacci Numbers**:

$$1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, \ldots$$

We use F_n to denote the n-th number in the list. For example, $F_1 = 1$, $F_2 = 1$, $F_3 = 2$, and $F_9 = 34$. In class (see your notes) we looked very carefully at ratios of successive terms, F_n/F_{n-1} and saw that the ratio approached the golden mean or ratio, ϕ.

a) Suppose we had computed the ratios upside-down instead and used F_{n-1}/F_n. Fill in the table below to see what happens.

F_{n-1}/F_n	1/1	1/2	2/3	3/5	5/8						
Decimal	1.0	0.5									

b) What number do the ratios appear to approach? Let's call it v. So $v \approx$ ____________________.

c) Compare v to the golden ratio, ϕ. Are they the same? If they are different, are they similar or related? Can you express v in terms of ϕ? (E.g., $v = 3\phi + 2$; this is not right, but can you find a very simple formula?)

2. The Ellen Sequence. Start a new sequence with two different numbers say: 4 and 1 and continue to generate new values—as in the Fibonacci sequence—by adding the previous two terms. We will name this new sequence the Ellen sequence (my wife). We will denote its terms by E_n.

a) Fill in the next few terms of the Ellen Sequence:

4	1									

b) Now fill in the ratios of adjacent terms, first as fractions, then as decimals:

Fraction	$\frac{1}{4}$								
Decimal	0.25								

c) Do the ratios seem to be approaching some number? Approximately what?

3. Try working with the Ellen ratios the same way that we worked with the Fibonacci and Gearan ratios. Use the previous ratio to determine. (See your class notes.)

$$\begin{aligned}
\frac{1}{4} &= \frac{1}{4} \\
\frac{5}{1} &= \frac{1+4}{1} = 1 + \frac{4}{1} = 1 + \frac{1}{\frac{1}{4}} \\
\frac{6}{5} &= \frac{5+1}{5} = 1 + \frac{1}{5} = 1 + \frac{1}{\frac{5}{1}} = 1 + \cfrac{1}{1 + \cfrac{1}{\frac{1}{4}}}
\end{aligned}$$

a) Write the next **two** ratios in a similar way. Do you see a pattern?

b) Suppose we keep going. Let the Greek letter *theta* (θ) represent the unending pattern. Write out the unending pattern for θ putting the fractional part in the box.

$$\theta = 1 + \cfrac{1}{\boxed{}}$$

c) What's in the box? Express the relation as an equation

$$\theta = 1 + \frac{1}{\boxed{}}$$

d) Solve for θ.

4. a) Solve for z in the equation below. Show your work.

$$z = \sqrt{3 + \sqrt{3 + \sqrt{3 + \sqrt{3 + \sqrt{3 + \cdots}}}}}$$

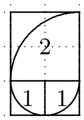
b) Solve for w in the equation below. Show your work.

$$w = 3 + \cfrac{1}{3 + \cfrac{1}{3 + \cfrac{1}{3 + \cfrac{1}{3 + \cfrac{1}{3 + \ddots}}}}}$$

5. Spiraling out of control. Try finishing the drawing on the next page. I started with a square measuring one unit across. Then I drew another square exactly the same size right beside it (so they are sharing a side). Then I took the line along the top of both squares and used that to draw a bigger square, which of course is two units on a side—right? So we have constructed squares with sides 1, 1, and 2 so far (recognize those numbers?). Okay, now moving clockwise around this construction, draw another square along the side where the edge of the big square and one small square align. Keep going clockwise and keep drawing bigger and bigger squares until your drawing extends beyond the grid.

a) Mark the edge length inside each square. List your lengths here: ________________________

b) Continue the clockwise curve that is drawn in the first three squares. Note that it goes from one corner to the diagonally opposite one. What is the shape of this curve?


2
1
1