Math 110: Practice for Test 1

- 1. The US State Department recognizes 194 independent countries. There are 7 continents.
 - a) If there are 25 ambassadors from different countries visiting the White House, what is the minimum number that must be from a single continent? Explain your answer.
 - b) What is the smallest number of ambassadors needed to ensure that there ALWAYS eight from at least one of the continents? Explain your answer.
- 2. a) The Gabonacci sequence (G_n) starts out as $G_1 = 3$ and $G_2 = 2$ and subsequent terms are sums of the previous two terms. What is G_{12} ? Show your work.
 - b) The Difonacci sequence (D_n) starts out as $D_1 = 1$ and $D_2 = 1$ and subsequent terms terms are sums are given by the **difference** of the previous two terms: $D_{n+1} = D_n D_{n-1}$ of the previous two terms. Write out the first 15 terms of the sequence. BE CAREFUL! What do you notice? What would D_{600} be? Explain.

D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9	D_{10}	D_{11}	D_{12}	D_{13}	D_{14}	D_{15}

- c) What is the Golden Ratio (exact value and decimal approximation)? What is its symbol? Briefly describe the process we used with Fibonacci numbers to determine the Golden Ratio.
- **3.** a) Suppose you start a game of Fibonacci nim with 251 sticks. What is your first move as player 1? Explain your strategy. If Player 2 plays the same number of sticks as you did, what would your next move be?
 - b) Suppose you are playing Fibonacci nim with another Math 110 student. You are Player 1. You start with 233 sticks. How much should you bet on the game? Explain.
- 4. a) Use your Sieve of Eratosthenes (just handed back) to find all the prime numbers between 90 and 110. Are there any twin primes in this interval?
 - b) What does the Goldbach conjecture say you should be able to do with the number 100? Can you do it? (Look at your sieve.)
 - c) Briefly explain what the first two passes through the Sieve of Eratosthenes do.
 - d) What does the infinitude of primes mean?
 - e) What does the Prime Factorization Theorem say?
- 5. a) Your uncle started college in the month of August. He took time off and went into the army and then worked for several additional years before finishing his degree. His graduation was 275 months after he started. What month was his graduation in? Briefly explain your answer expressing your work in modular arithmetic.
 - b) The following is the UPC for a box of Kashi Stoneground 7 Grain Crackers: 0-18d27-61006-9. What is the missing digit? Show your work and explain your answer.
- 6. Determine the following equivalents.
 - a) $59 \times 24 \pmod{9}$. Show your work.
 - **b)** $5^{500} \pmod{24}$. Show your work.
 - c) $5^{501} \pmod{24}$. Show your work.
 - d) $6^{103} \pmod{36}$. Show your work.
 - e) Recall Fermat's Little Theorem: If p is a prime number and n is any integer that does not have p as a factor, then $n^{p-1} \equiv 1 \pmod{p}$.
 - f) Use Fermat's Little Theorem to determine: $5^{16} \pmod{17}$. (Show your work.)
 - g) Now determine: $5^{19} \pmod{17}$. (Show your work.)
 - h) Use Fermat's Little Theorem to determine: 3¹⁵ (mod 13). (Show your work.)

7. Solve for w in the equation below. Show your work.



- 8. a) Suppose m and n are natural numbers. What does the Division Algorithm say about these numbers?
 - b) Consider the number $m = (1 \times 2 \times 3 \times 4 \times 5 \times 6) + 1$. Give a brief explanation of why m has no factors smaller than 7 (other than 1).
 - c) Evaluate the above number $m \pmod{3}$. Quickly calculate $m \pmod{2}$, $m \pmod{4}$, $m \pmod{5}$, and m(mod 6). How are your answers related to the previous part?
- 9. a) A number m is divided by 36 and has a remainder of 27. Express m using the division algorithm.
 - b) Take this same number m and add 213 to it. What is the remainder when this new number is divided by 9?
 - c) Using the same m, what is $m + 213 \pmod{9}$?
- **10.** a) True or false: If n is a factor of m, then $m \equiv 0 \pmod{n}$. Explain your answer. **b)** Is $84 \equiv 36 \pmod{24}$? Explain.
- 11. What is a conjecture? What is the Goldbach conjecture? The twin prime conjecture?